Staple!

Team: V

Sets and Logic
MHF3202 2787

Home-V

Prof. JLF King
Wednesday, 23Oct2019

Due **BoC, Monday, 28Oct2019**, wATMP!

Please *fill-in* every *blank* on this sheet. Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0 \neq$ *Empty-word*.

V1: *Show no work.*

a Sequence $\vec{\mathbf{L}} := (L_n)_{n=0}^{\infty}$ is defined by $L_0 := 0$, $L_1 := 1$, and $\boxed{\forall n \in \mathbb{N}:\ L_{n+2} = 3L_{n+1} + L_n}$. This implies $\boxed{\forall k \in \mathbb{N}:\ L_k = [P \cdot \alpha^k + Q \cdot \beta^k]}$, for real numbers $\alpha=$ ________ $> \beta=$ ________, $P=$ ________, $Q=$ ________.

b $\mathbf{Q} := \{(6,2), (2,7), (5,2), (7,3), (7,5), (3,4), (1,3), (1,5)\}$ is a binrel on $[1..7]$, with transitive closure $\mathbf{R}$. Then:

2**R**2 is *T F*. 4**R**6 is *T F*. 7**R**7 is *T F*.

c *Shuffling 2N-card deck*: Put the upper N in your right hand, and the lower N in left hand. Drop a RH card, then a LH, then RH, etc. [New bottom-card came from RH; new top-card from LH.] So $S_N := \text{Sign}(\pi_N) =$ ________

And S_{2019} *is* circle **+1** **−1**.

d Rewrite $x \in \bigcup_{j \in A}[G_j \cup [\bigcap_{k \in B} H_k]]$ without $\cup, \cap$, only using $\boxed{\forall\ :\ \exists\ \text{st.}\ \in\ \ni\ \vee\ \wedge\ ]\ [\ x\ G\ H\ k\ j\ A\ B}$,

as ____.

For the two essay questions, carefully TYPE, *double spaced, grammatical solns.*

V2: Let $\mathbf{E}_n$ be the equilateral triangle with side-length 2^n. This $\mathbf{E}_n$ can be tiled in an obvious way by 4^n many little-triangles [copies of $\mathbf{E}_0$]; [picture on blackboard] The **"*punctured* $\mathbf{E}_n$"**, written $\widetilde{\mathbf{E}_n}$, has its topmost copy of $\mathbf{E}_0$ removed.

A ***(trape)zoid***, $\mathbf{T}$, comprises three copies of $\mathbf{E}_0$ glued together in a row, rightside-up, upside-down, rightside-up [picture on blackboard] [A ***zoid-tiling*** allows all three rotations of $\mathbf{T}$.]

i PROVE: *For each n, board $\widetilde{\mathbf{E}_n}$ admits a zoid-tiling.*

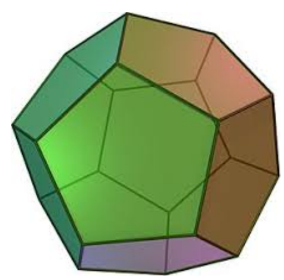
ii Let Δ_k be the equilateral triangle of sidelength k; so $\mathbf{E}_n$ is Δ_{2^n}. Triangle Δ_k comprises k^2 little-triangles.
For what values of k does Δ_k admit a zoid-tiling?
For which k does $\widetilde{\Delta_k}$ admit a zoid-tiling?

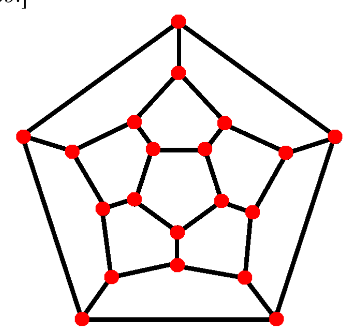
V3: [A dodecahedron is a regular polyhedron having 12 faces, 20 vertices and 30 edges; the faces are pentagons.] Two vertices of a regular dodecahedron are ***cousins*** if they are *distinct* vertices of a common face. [Each vertex has $[3 \cdot 4] - 3 = 9$ cousins.] Write $v \sim w$ to indicate that v and w are cousins. Easily, $\sim$ is symmetric, and anti-reflexive. You can check that $\sim$ is not transitive.

A ***labeling*** of a regular dodecahedron assigns, to each vertex, a *positive integer*. A labeling is ***legal*** IFF *no* pair $v \sim w$ of vertices is assigned the same label.

i Prove there is no legal labeling with vertex-sum [the sum of the 20 labels] equaling 59.

ii Let $\mathcal{S} \subset \mathbb{Z}_+$ be the *set* of vertex-sums obtainable from legal-labelings. Characterize $\mathcal{S}$ explicitly, with proof. You will likely need to construct some particular legal-labelings. [You showed, above, that $\mathcal{S} \not\ni 59$.]





V1:	___ ___ ___	130pts
V2:	___ ___ ___	115pts
V3:	___ ___	95pts
Total:	___ ___ ___	340pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* Name/Signature/Ord

Ord: ____

Ord: ____

Ord: ____