Staple!

Team V____

Sets and Logic
MHF3202 8768

Home-V

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Due **BoC, Mon., 30Jan2017**, wATMP! Please *fill-in* every *blank* on this sheet. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

V1: *Show no work. Simply fill-in each blank on the problem-sheet.*

a Using *only* symbols $\boldsymbol{H}, \boldsymbol{D}, \wedge, \vee, \neg, T, F,], [$, rewrite (in simplest form) expression $[[H\Rightarrow D]\Rightarrow H]$ as ________________. Ditto, rewrite $[H\Rightarrow[D\Rightarrow H]]$ as ________________.

b Write the free vars in each of these expressions.

$$\overbrace{\exists n \in \mathbb{N}\colon\ f(n) \subset \underbrace{\bigcup_{\ell=r-4}^{r+7} \underbrace{\{x\in\mathbb{Z} \mid \ell\cdot n \equiv_5 x^2\}}_{E1}}_{E2}}^{E3}$$

$E1$: ________. $E2$: ________. $E3$: ________.

c The coeff of x^7y^{12} in $[5x+y^3+1]^{30}$ is ________________.

d $\forall x,z \in \mathbb{Z}$ *with* $x<z$, $\exists y \in \mathbb{Z}$ *st.:* $x<y<z$. T F

$\forall x,z \in \mathbb{Q}$ *with* $x \neq z$, $\exists y \in \mathbb{R}$ *st.:* $x<y<z$. T F

For all sets Ω, *there exists a fnc* $f\colon\mathbb{R}\to\Omega$. T F

For the three essay questions, carefully TYPE *triple–spaced, grammatical, solns. I suggest* LaTeX, *but other systems are ok too.*

V2: Kindergarten teacher Mrs. Nice has an unlimited supply of cookies. With her N pupils sitting in a circle, Mrs. Nice stands in front of *Abby*. Circling clockwise she takes one step, gives that student, *Bert*, a cookie. Continuing, she takes two steps (passing *Carol*), giving *Danny* a cookie. Then three steps, (passing *Englebert* and *Fran*), giving *Gail* a cookie. Four steps, cookie, five steps, cookie, and so on. [Some kids are getting lots of cookies.]

i Prove that if $N\geqslant 3$ is *prime*, then some child **never** gets a cookie. [*Hint:* PHP.]

ii What can you tell me: *When N is even? When N is a non-prime odd? When N has form 2^k? 3^k?* For what values of N does *Abby* get at least one cookie?

Let $V(N)$ be the number of kids who *neVer* get a cookie. How large can $V(N)$ get? [The above asked you to prove that $V(\text{OddPrime}) \geqslant 1$.] How large does ratio $\frac{V(N)}{N}$ get? What conjectures/theorems can you come up with? If you know how to program, do some computer experiments.

V3: Define a sequence $\vec{\mathbf{b}} = (b_0, b_1, b_2, \ldots)$ by $b_0 := 0$ and $b_1 := 3$ and

$$\dagger\colon\qquad b_{n+2} := 7b_{n+1} - 10b_n\,, \quad \text{for } n = 0,1,\ldots.$$

Use induction to prove, for each natnum k, that

$$\ddagger\colon\qquad b_k = 5^k - 2^k\,.$$

Further: Given recurrence (†) and initial conditions, *explain* how you could have discovered/computed the numbers 5 and 2 in the (‡) formula.

Can you generalize to getting a (‡)-like formula when the recurrence is $\boxed{b_{n+2} := \mathbf{S}b_{n+1} - \mathbf{P}b_n}$, for arbitrary real-number coefficients $\mathbf{S}$ and $\mathbf{P}$?

V4: Note $f(n) := \frac{1}{2}\cdot[5^n + 21^n]$ is an integer. Prove, for each *odd* $n\geqslant 3$, that $f(n)$ is composite. [*Hint:* Look at $f(n)$ mod something.]

[Are all the hypotheses necessary, or can some be weakened?]

End of Home-V

V1:	___ ___ ___	110pts
V2:	___ ___	80pts
V3:	___ ___	70pts
V4:	___ ___	65pts
Total:	___ ___ ___	325pts