

Due **BoC, Monday, 21Oct2013.**, Please *fill-in* every *blank* on this sheet. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

**V1:** *Show no work, here. Simply fill-in each blank on the problem-sheet.*

**a** Sequence  $\vec{L} := (L_n)_{n=0}^\infty$  is defined by  $L_0 := 0$ ,  $L_1 := 1$ , and  $\forall n \in \mathbb{N}: L_{n+2} = 3L_{n+1} + L_n$ . This implies  $\forall k \in \mathbb{N}: L_k = [P \cdot \alpha^k + Q \cdot \beta^k]$ , for real numbers  $\alpha = \dots > \beta = \dots$ ,  $P = \dots$ ,  $Q = \dots$ .

**b** Compute the real  $\alpha = \dots$  such that  $\dots$

$$3^\alpha \cdot \sum_{k=0}^{4004} \binom{4004}{k} 2^k = \sum_{j=0}^{2013} \binom{2013}{j} 8^j.$$

[Hint: The Binomial Theorem]

**c** The number of ways of having 4 objects from 9 types is  $\left[ \begin{matrix} 4 \\ 9 \end{matrix} \right] \frac{\text{Binom}}{\text{coeff}} \left( \dots \right) \frac{\text{Integer}}{\text{numeral}} \dots$

And  $\left[ \begin{matrix} 4 \\ 9 \end{matrix} \right] = \left[ \begin{matrix} K \\ L \end{matrix} \right]$ , where  $K = \dots \neq 4$ , and  $L = \dots$

**d** On  $\Omega := [1..29] \times [1..29]$ , define binary-relation **C** by:  $(x, \alpha) \mathbf{C} (y, \beta)$  IFF  $x \cdot \beta \equiv_{30} y \cdot \alpha$ . Statement "Relation **C** is an **equivalence relation**" is:  $T \quad F$

Your 2 essay(s) must be TYPESET, and Double or Triple spaced. Use the Print/Revise cycle to produce good, well thought out, essays. Start each essay on a **NEW** sheet of paper. Do **not** restate the problem; just solve it.

**V2:** On a 9x9 chessboard, 37 rooks are placed. Prove there exists a **friendly** 5-set of rooks. [I.e, on 5 distinct rows and on 5 distinct columns.] [Hint: PHP] Illustrate the concepts in your proof with large, useful Pictures.

**V3:** For all natnums  $k < n$ , prove that  $H_k \perp H_n$ , where

$$H_k := 1 + 6^{[2^k]}.$$

[Hint: For each natnum  $m$ , define  $G_m := -1 + 6^{[2^m]}$ . Prove a divisibility relation among the  $H$ s and the  $G$ s, by induction. Then a common divisor of  $H_k$  and  $H_n$  must...]

Also, produce an index  $\ell \in \mathbb{N}$  st.  $H_\ell$  is not prime.

End of Home-V

<b>V1:</b>	_____	95pts
<b>V2:</b>	_____	95pts
<b>V3:</b>	_____	105pts
<b>Total:</b>	_____	295pts

**HONOR CODE:** "I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)." Name/Signature/Ord

Ord: \_\_\_\_\_

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