Staple!

Team V____

Sets and Logic MHF3202 2787 — **Home-V** — Prof. JLF King, *Touch:* 4Oct2017

Due **BoC, Monday, 21Oct2013.**, Please *fill-in* every *blank* on this sheet. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

V1: *Show no work here. Simply fill-in each blank on the problem-sheet.*

___ ___ **a** Sequence $\vec{\mathbf{L}} := (L_n)_{n=0}^{\infty}$ is defined by $L_0 := 0$, $L_1 := 1$, and $\forall n \in \mathbb{N}\colon\ L_{n+2} = 3L_{n+1} + L_n$. This implies $\forall k \in \mathbb{N}\colon\ L_k = [P{\cdot}\alpha^k + Q{\cdot}\beta^k]$, for real numbers $\alpha=$ ________ $> \beta=$ ________, $P=$ ________, $Q=$ ________.

___ ___ **b** Compute the real $\alpha=$ ________________ such that

$$*:\qquad 3^{\alpha}\cdot\sum_{k=0}^{4004}\binom{4004}{k}2^k \;=\; \sum_{j=0}^{2013}\binom{2013}{j}8^j\,.$$

[*Hint:* The Binomial Theorem]

___ ___ **c** The number of ways of having 4 objects from 9 types is $\left[\!\left[\begin{matrix}4\\9\end{matrix}\right]\!\right] \overset{\text{Binom}}{\underset{\text{coeff}}{=}} \binom{\quad}{\quad} \overset{\text{Integer}}{\underset{\text{numeral}}{=}}$ ________________.

And $\left[\!\left[\begin{smallmatrix}4\\9\end{smallmatrix}\right]\!\right] = \left[\!\left[\begin{smallmatrix}N\\T\end{smallmatrix}\right]\!\right]$, where $N=$ ________ $\neq 4$, and $T=$ ________.

___ ___ **d** On $\Omega := [1\,..\,29]\times[1\,..\,29]$, define binary-relation $\mathbf{C}$ by: $(x,\alpha)\,\mathbf{C}\,(y,\beta)$ IFF $x{\cdot}\beta \equiv_{30} y{\cdot}\alpha$. Statement "*Relation* **C** *is an* ***equivalence relation***" is: *T* *F*

Your 2 essay(s) must be TYPESET, *and Double or Triple spaced. Use the* Print/Revise *cycle to produce good, well thought out, essays. Start each essay on a* ***new*** *sheet of paper. Do* **not** *restate the problem; just solve it.*

V2: On a 9×9 chessboard, 37 rooks are placed. Prove there exists a ***friendly*** 5-set of rooks. [I.e, on 5 distinct rows and on 5 distinct columns.] [*Hint:* PHP] Illustrate the concepts in your proof with *large*, *useful* Pictures.

V3: For all natnums $k < n$, prove that $H_k \perp H_n$, where

$$H_k \;:=\; 1 + 6^{[2^k]}\,.$$

[*Hint:* For each natnum m, define $G_m := -1 + 6^{[2^m]}$. Prove a divisibility relation among the Hs and the Gs, by induction. Then a common divisor of H_k and H_n must...]

Also, produce an index $\ell\in\mathbb{N}$ st. H_ℓ is not prime.

End of Home-V

V1:	___ ___	95pts
V2:	___ ___	95pts
V3:	___ ___ ___	105pts
Total:	___ ___ ___	295pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* Name/Signature/Ord

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