Staple!

Ord: __ __

Differential Eqns MAP2302 **V-IOP** Prof. JLF King **v.2**

This TYPED IOP (Individual Optional Project) is due **2PM, Thur., 06Dec2018**, slid *completely* under my office door, 402 Little Hall. *This sheet* is "Page 1/N"; you've labeled the rest as "2/N",..., "N/N".

Your 4 essay(s) must be TYPED, and Double spaced. Use the *Print/Revise* cycle to produce good, well thought out, essays. Start each essay on a *new* sheet.

V1: i Suppose that $P(), Q(), G()$ are $\mathbf{C}^\infty$-fncs, and $y = y(t)$ is a soln to DE

$$**: \qquad y'' + P(t)y' + Q(t)y = G(t).$$

Rigorously argue that y **must** be a $\mathbf{C}^\infty$-fnc.

ii Consider an analytic soln $y = \sum_{n=0}^\infty b_n t^n$ to IVP

$$\dagger: \qquad \begin{aligned} &y'' + y' + y = \cos(10\,t)\,, \\ &\textit{with } y(0) = 0 \textit{ and } y'(0) = 1. \end{aligned}$$

Writing $\cos(10\,t) = \sum_{n=0}^\infty C_n t^n$, derive a recurrence relation for seq $\vec{\mathbf{b}} = (b_0, b_1, \dots)$:

$b_{n+2} =$ ....

[The blank will be ITOf b_n, b_{n+1} and appropriate terms from seq $C_0, C_1, \dots$.] Show how to use it, or another method, to compute the first three *non-zero* b-terms. Write your answers in $b_? = ?$ form.

V2: Consider U.F $y = y(t)$ satisfying IVP

$$*: \qquad ty'' + y' + ty \quad = \quad 0\,,$$

where $y(0) = 1$ and $y'(0) = 0$. Let $P(s) := \big[\mathcal{L}(y)\big](s)$ denote the La*P*lace transform of y.

a Use our properties of $\mathcal{L}()$ to derive a *first-order* separable DE that $P()$ satisfies.

b Use SoV to solve for $P()$. [Compute the integral.]

V3: Solve problem #**36**$^{P.}$*391*.NSS9, the **unit triangular pulse** problem, in a carefully written essay. Also include good pictures in your explanation.

V4: UFs $x(t)$ and $y(t)$ satisfy differential system

E1a: $x' - 6x + 3y = 8e^t$, *and*

E1b: $y' - 2x - y = 4e^t$,

with *initial conditions*

E2: $x_0 := x(0) = 1$ *and* $y_0 := y(0) = 0$.

Solve this system using the Laplace xform. Here, use X and Y to denote the xforms of x and y. Write $f(t) := 8e^t$ and $h(t) := 4e^t$, with $F(s)$ and $H(s)$ their xforms. Use the following ideas to solve the system.

Step 1. Compute the xforms of (E1a,b), using (E2). [Justify your steps with the appropriate thms.] This will give two algebraic eqns, each ITOf X and Y.

Step 2. Solve the algebraic system to get two xform formulae $X = \frac{\mathcal{A}}{\mathcal{R}}$ and $Y = \frac{\mathcal{B}}{\mathcal{R}}$, where

$\mathcal{A}(s)=$ ...., $\mathcal{B}(s)=$ ....,

$\mathcal{R}(s)=$ ....

are **polynomials**. *Write $\mathcal{R}(s)$ as a product of degree-1 polynomials. I.e, $\mathcal{R}(s)$ is fully factored.*

Step 3. [*Short answer*] The inverse-xforms are

$x(t) =$ ....,

$y(t) =$ ....

[Use a CA-system; don't show the computation. *For yourself*, verify that your x and y fulfill (E1a,b) & (E2).]

Print name .... Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....

Folks, we've learned a lot of DfyStuff this semester, and I've had a great time with a lively (and funny) *class of Enthusiastic DiffyQers. Stop by in future semesters for Math/chess/coffee/frisbee...*

Cheers, Prof. K