Staple!

Ord: ___ ___

Differential Eqns MAP2302 | **V-IOP** | Prof. JLF King Thur, 19Apr2018

This TYPED IOP (Individual Optional Project) is due **2PM, Thur., 26Apr2018**, slid *completely* under my office door, 402 Little Hall. *This sheet* is "Page 1/*N*"; you've labeled the rest as "2/*N*",..., "*N*/*N*".

Your 3 essay(s) must be TYPED, and Double or Triple spaced. Use the *Print/Revise* cycle to produce good, well thought out, essays. Start each essay on a ***new*** sheet.

V1: We have 4 brine-filled tanks. Call the ocean "tank-0", with salinity $\mathbf{S_0} :: \frac{\mathsf{lb}}{\mathsf{gal}}$. For $j = 1, \ldots, 4$, the *initial* salinity in tank-j is $S_j :: \frac{\mathsf{lb}}{\mathsf{gal}}$. The *initial* volume in each tank is $\mathsf{U} := 50\mathsf{gal}$.

For $j = 0, \ldots, 4$, tank-j outputs into tank-$[j{+}1]$ ("tank-5" is the drain), at rate $[10 - j]\frac{\mathsf{gal}}{\mathsf{min}}$.

Let $\sigma_j(t)$ denote the salinity in tank-j at time t. With Large, Labelled, carefully drawn pictures, show the situation and carefully derive a DE for each $\sigma_j()$. Carefully define any additional fncs you need (such as the volume of water in a tank at a given time) so as to make your derivation clear and rigorous.

Solve the DEs as far as you can, using any of our methods from class, with explanation.

If you are unable to get exact solns for some DEs, give numerical approximations.

V2: i Suppose that $P(), Q(), G()$ are $\mathbf{C}^\infty$-fncs, and $y = y(t)$ is a soln to DE

$$*: \qquad y'' + P(t)y' + Q(t)y \;=\; G(t)\,.$$

Argue carefully that y *must* be a $\mathbf{C}^\infty$-fnc.

ii Assume that there is an analytic soln $y = y(t)$ to IVP

$$\dagger: \qquad y'' + y' + y \;=\; \cos(10\,t)\,, \quad \textit{with}\;\; y(0) = 0 \;\; \textit{and} \;\; y'(0) = 1.$$

Compute the first three *non-zero* terms of the Taylor polynomial approximation [centered at 0] of $y(t)$.

V3: Solve problem #**36**$^{P.391}$.NSS9, the **unit triangular pulse** problem, in a carefully written essay. Also include good pictures in your explanation.

V4: *Short answer.*

a $\mathbb{R}$-matrices $\mathsf{U}, \mathsf{G}, \mathsf{R}$ are 3×3, with U invertible and R nilpotent. [Use $\mathbf{I}$ for the 3×3 identity matrix.]

Matrix URU^{-1} *is nilpotent*:	*AT*	*AF*	*Nei*
Each entry of e^{tR} *is a polynomial*:	*AT*	*AF*	*Nei*
Matrix e^{R} *is nilpotent*:	*AT*	*AF*	*Nei*
R^2 *is the zero-matrix*:	*AT*	*AF*	*Nei*
Matrix $\mathsf{e}^{[G+I]G}$ *equals* $\mathsf{e}^{G} \cdot \mathsf{e}^{G^2}$:	*AT*	*AF*	*Nei*
Matrix $\mathsf{e}^{[G^2]}$ *equals* $[\mathsf{e}^{G}]^2$:	*AT*	*AF*	*Nei*

End of V-IOP

Print name . . . . . Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: . . . . . .

Folks, we've learned a lot of DfyStuff this semester, and I've had a great time with a lively (and funny) class of Enthusiastic DiffyQers. Stop by in future semesters for Math/chess/coffee/frisbee...

Cheers, Prof. K