

V4: Short answer. Show no work.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a Sequence $\vec{L} := (L_n)_{n=0}^\infty$ is defined by $L_0 := 5$, $L_1 := 4$, and $\forall n \in \mathbb{N}: L_{n+2} = 3L_{n+1} + 2L_n$.

This implies $\forall k \in \mathbb{N}: L_k = [P \cdot \alpha^k + Q \cdot \beta^k]$, for real numbers

$\alpha = \dots < \beta = \dots$

b The physics lab has atomic *zinc, tin, silver* and *gold*. I'm allowed to take 5 atoms, so I have [expressed as single integer] \dots many possibilities.

c Mimicking what we did in class: From the 300×200 game-board, cut-out (remove) the **(35, 150)**-cell and one other cell at $P = (x, y)$. **Circle** those choices for P ,

(150, 160), (14, 35), (66, 77), (195, 15), (123, 4)

which, if removed, would leave a board that *definitely cannot* be domino-tiled.

d On \mathbb{R}_+ , define several relations: Say that $x \mathcal{R} y$ IFF $y - x < 17$. Define \mathcal{P} by: $x \mathcal{P} y$ IFF $x^{\log(y)} = 5$. Say that $x \mathcal{I} y$ IFF $x + y$ is irrational.

Use \bullet for the "divides" relation on the positive integers: $k \bullet n$ iff there exists a positive r with $rk = n$.

d1 Please **circle** those of the following relations that are *transitive* (on their domain of defn).

$\neq \bullet \leq \mathcal{R} \mathcal{P} \mathcal{I}$

d2 **Circle** the *symmetric* relations:

$\neq \bullet \leq \mathcal{R} \mathcal{P} \mathcal{I}$

d3 **Circle** the *reflexive* relations:

$\neq \bullet \leq \mathcal{R} \mathcal{P} \mathcal{I}$

OYOP: In grammatical English *sentences*, write your essay on every *third* line (usually), so that I can easily write between the lines. Do **not** restate the question.

V5: On a 7×7 chessboard, 23 rooks are placed. Prove: **Either** there exists a friendly 5-set, or disjoint-pair of friendly 4-sets. [An n -set of rooks is **friendly** if the rooks lie on n distinct rows, and n distinct columns.] [Hint: PHP]

End of Class-V

V4: _____ 75pts

V5: _____ 75pts

Total: _____ 150pts

Please PRINT your *name* and *ordinal*. Ta:

_____ Ord: _____

HONOR CODE: "I have neither requested nor received help on this exam other than from my professor."

Signature: _____