Staple!

Ord: ___ ___

Plex MAA4402 8436 | **Class-Prac-V** | Prof. JLF King 04Oct2021

Notation. All sets are subsets of $\mathbb{C}$.
For sets B and E, the difference set is

$$B \setminus E \quad := \quad \left\{x \in B \,\middle|\, x \notin E\right\}.$$

The complement of E is $E^c := \mathbb{C} \setminus E$.
For short-answer: Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$.

Prac-V1: Short answer. Show no work. $\mathbb{C}$-plane

a) Blanks$\in\mathbb{R}$. So $\frac{1}{2+3\mathbf{i}} =$ ______ $+\mathbf{i}\cdot$ [______].

Thus $\dfrac{5-\mathbf{i}}{2+3\mathbf{i}} =$ ______ $+\mathbf{i}\cdot$ [______].

For z complex, $\mathrm{Im}(z) = \mathrm{Formula}(z, \overline{z}) =$ ______.

b) Complex number $[x + \mathbf{i}y]^2 = -9\mathbf{i}$, for *real numbers* $x>y$, where $x=$ $+\frac{3}{\sqrt{2}}$ and $y=$ $-\frac{3}{\sqrt{2}}$.

Sqroot: Since $-\mathbf{i} = \exp(\mathbf{i}\cdot\frac{-\pi}{2})$, one sqroot of $-\mathbf{i}$ is $\exp(\mathbf{i}\cdot\frac{-\pi}{4}) \overset{\text{note}}{=\!=} \frac{1-\mathbf{i}}{\sqrt{2}}$. Multiplying by $\sqrt{9} = 3$ gives a particular sqroot of $-9\mathbf{i}$, namely $\frac{3}{\sqrt{2}}\cdot[1 - \mathbf{i}]$.

c) *Reals* $x=$ ______ and $y=$ ______ where $x + \mathbf{i}y = [1+\mathbf{i}]^{166}$. [*Hint: Multiplying complexes multiplies their moduli, and adds their angles.*]

d) Distance $\left|e^{\mathbf{i}[\pi/4]} - 2\mathbf{i}\right| =$ ______.

Soln: With $R := \frac{1}{\sqrt{2}}$ [Reciprocal], note $e^{\mathbf{i}\cdot\frac{\pi}{4}} = [1+\mathbf{i}]R$. With $\mathbf{D} := \left|[1+\mathbf{i}]R - 2\mathbf{i}\right|$, then $\mathbf{D}^2$ equals

$$\begin{aligned}[R-0]^2 + [R-2]^2 &= R^2 + \left[R^2 + 4 - 4R\right] \\ &= 1 + 4 - 4R = 5 - 2\sqrt{2}\,.\end{aligned}$$

Thus, $\mathbf{D} = \sqrt{5 - 2\sqrt{2}}$.

e) Value $\left|e^{\mathbf{i}\theta} - \mathbf{i}\right| = 2$, where angle $\theta=$ ______.

f) The ***Laplacian*** of a twice-differentiable fnc $h{:}\,\mathbb{R}\times\mathbb{R}\to\mathbb{R}$, is ______.

g) Fnc $u(x,y) := \cos(y\cdot x) - 7x$ maps $\mathbb{R}\times\mathbb{R}\to\mathbb{R}$. Its Laplacian is $[\Delta(u)](x,y) =$ ______.

There exists function $v{:}\,\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ such that $f(x+\mathbf{i}y) := u(x,y) + \mathbf{i}v(x,y)$ is holomorphic. *T* *F*

h The std.pic. of $\mathbb{C}$ is called the ________ plane.

Set $w := \log(3)\cdot[2+\mathbf{i}]$. So $|w|=$ ________.

$\text{Re}(e^w)=$ ________ and $|e^w|=$ ________.

i Write $\cos(-2\mathbf{i})$, which is real, ITOf exp() and *finite* add/sub/mul/div: $\cos(-2\mathbf{i})=$ ________.

And $\cos(-2\mathbf{i})$ lies in [circle the correct interval]

$(-\infty, \frac{-1}{5}]$ $(\frac{-1}{5}, \frac{1}{5}]$ $(\frac{1}{5}, 2]$ $(2, 5]$ $(5, 15]$ $(15, 45]$ $(45, \infty)$

j Value $\text{Log}([\mathbf{i}e]^3)=$ ________.

[P.V of $[1+\mathbf{i}]^{\mathbf{i}}$] $= r\cdot\exp(\mathbf{i}\theta)$, where $r=$ ________ and $\theta=$ ________, with $r>0$ and θ real.

Prac-V2: Short answer. Metric space stuff

k On a set Ω, a ***metric*** is a map $\mathsf{d}:\Omega\to[0,\infty)$ such that $\forall\, w,x,y,z \in \Omega$:

MS1: ________.

MS2: ________.

MS3: ________.

l A subset $S \subset \mathbb{C}$ is ***path-connected*** if ________.

... if for *all* points $p,q\in S$, *there exists* a map $f:[0,1]\to S$ which is **continuous**, with $f(0)=p$ and $f(1)=q$.

m

The empty-set is connected:	*T*	*F*
Punctured ball $\text{PBal}_2(3\boldsymbol{i})$ *is connected:*	*T*	*F*
$\text{Sph}_2(5\boldsymbol{i}) \cap \text{Sph}_2(\boldsymbol{i})$ *is connected:*	*T*	*F*
$\text{Sph}_2(4\boldsymbol{i}) \cup \text{Sph}_2(-\boldsymbol{i})$ *is connected:*	*T*	*F*
$\text{Sph}_2(5\boldsymbol{i}) \cup \text{CldBal}_2(\boldsymbol{i})$ *is closed:*	*T*	*F*

n All these sets are non-empty: Sets U and V are open. Sets K, E and E_n are closed. Sets S and T are each connected.

Set $U \smallsetminus K$ *is open:*	*AT*	*AF*	*Nei*
Set $U \cup K$ *is open:*	*AT*	*AF*	*Nei*
Set $E \cap K$ *is closed:*	*AT*	*AF*	*Nei*
Union $\bigcup_{n=1}^{\infty} E_n$ *is closed:*	*AT*	*AF*	*Nei*
$\exists q \in [S \cap T]$; *so* $S \cup T$ *is connected:*	*AT*	*AF*	*Nei*

o Let $S := \text{PBal}_2(3\mathbf{i})$.

Its boundary is $\partial(S) =$ ________.

[You may use our ball/sphere notation as well as $\cup$, $\cap$, complement and set-braces, to describe your answer.]