Staple!

Team: U ___

Sets and Logic
MHF3202 2787

Home-U

Prof. JLF King
Thursday, 20Sep2018

Due **BoC, Monday, 23Sep2019**, wATMP!
Please *fill-in* every *blank* on this sheet. Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0 \neq$ *Empty-word*.

U1: *Show no work. Simply fill-in each blank on the problem-sheet.*

__ __ **a** Given sets with cardinalities $|B| = 8$ and $|E| = 5$, the number of non-constant fncs in B^E is ________.

__ __ **b** Using *only* symbols $\boldsymbol{H}$, $\boldsymbol{D}$, $\wedge$, $\vee$, $\neg$, T, F,], [, rewrite (in simplest form) expression $[[H \Rightarrow D] \Rightarrow H]$ as ________. Ditto, rewrite $[H \Rightarrow [D \Rightarrow H]]$ as ________.

__ __ **c** $\forall x,z \in \mathbb{Z}$ *with* $x < z$, $\exists y \in \mathbb{Z}$ *st.:* $x < y < z$. T F
$\forall x,z \in \mathbb{Q}$ *with* $x \neq z$, $\exists y \in \mathbb{R}$ *st.:* $x < y < z$. T F
For all sets Ω*, there exists a fnc* $f:\mathbb{R} \to \Omega$. T F

__ __ **d** The coeff of $x^7 y^{12}$ in $[5x + y^3 + 1]^{30}$ is ________.
[You may write in form number times multinomial-coeff. You can leave the multinomial-coeff as such, or write ITOf factorials.]

__ __ **e** The number of ways of picking 42 objects from 70 types is $\left[\!\left[\begin{matrix}70\\42\end{matrix}\right]\!\right] \stackrel{\text{Binom}}{\underset{\text{coeff}}{=}} \Big(\_\_\_\_\_\_ \Big)$. And $\left[\!\left[\begin{smallmatrix}70\\42\end{smallmatrix}\right]\!\right] = \left[\!\left[\begin{smallmatrix}T\\N\end{smallmatrix}\right]\!\right]$, where $T=$ ______ $\neq 70$, and $N=$ ______.

For the two essay questions, carefully TYPE*, double spaced, grammatical solns. I suggest* LaTeX*, but other systems are ok too.*

U2: Give a careful bijective proof of:

Thm: *Fix a natnum* $N \geqslant 3$*. Then*

$$*: \quad [\![N \downarrow 3]\!] \cdot 2^{N-3} = \sum_{k=3}^{N} [\![k \downarrow 3]\!] \cdot \binom{N}{k}.$$

[Can you also prove this by induction on N?]

U3: **i** An ***Lmino*** (pron. "ell-mino") comprises three  squares in an "L" shape (all four orientations are allowed).

Let $\mathbf{S}_n$ be the $2^n \times 2^n$ square board, comprising 4^n ***squaries*** (little squares). Have $\widetilde{\mathbf{S}_n}$ be the board with one corner squarie removed. We showed in class that that each $\widetilde{\mathbf{S}_n}$ is Lmino-tilable (by $[4^n - 1]/3$ Lminos, of course). Further, with $\mathbf{S}'_n$ denoting $\mathbf{S}_n$ with an *arbitrary* puncture, we proved that every $\mathbf{S}'_n$ is Lmino-tilable.

Generalize this to three-dimensions. Let $\mathbf{C}_n$ denote the $2^n \times 2^n \times 2^n$ cube, $\widetilde{\mathbf{C}_n}$ the corner-punctured cube, and let $\mathbf{C}'_n$ be $\mathbf{C}_n$ but with an arbitrary ***cubie*** removed.

What is the 3-dimensional analog of an Lmino? Calling it a **"3-*mino*"**, how many cubies does it have? [Provide a drawing of your 3-mino.] PROVE: *Every* $\mathbf{C}'_n$ *admits a 3-mino–tiling.* [Provide also pictures showing your ideas.]

ii Generalize to K-dim(ensional) space, with $\mathbf{C}_{n,K}$ being the $2^n \times \overset{K}{\cdots} \times 2^n$ cube, having $[2^n]^K = 2^{nK}$ many K-dim'al cubies. As before, let $\mathbf{C}'_{n,K}$ be $\mathbf{C}_{n,K}$ with an arbitrary cubie removed.

What is your K***-mino*** *with which you will tile, and how many cubies does it have?* (So a 2-mino is our Lmino.) PROVE: *Every* $\mathbf{C}'_{n,K}$ *admits a* K*-mino–tiling.*

U4: Pick/create a non-trivial PHP problem, then give a nice soln. Extra credit if there is a clever visual soln.

End of Home-U

U1:	__ __ __	135pts
U2:	__ __	65pts
U3:	__ __ __	105pts
U4:	__ __	25pts
Total:	__ __ __	330pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

________________ Ord: __ __

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