Sets and Logic MHF3202 09EH — **Class-U** — Prof. JLF King, Wednesday, 25Sep2019

U5: Short answer. Show no work.
Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0 \neq$ *Empty-word*.

__ __ **a** Prof. King believes that writing in complete, coherent sentences is crucial in communicating Mathematics, improves posture, and whitens teeth. Circle one:

True! **Yes!** ***wH'at S a?sEnTENcE***

__ __ **b** For a finite list $\mathcal{S}$ of posints, define

$$\boldsymbol{\mu}_{\mathcal{S}}(N) := \left\{ k \in [1..N] \;\middle|\; \exists d \in \mathcal{S} \text{ with } d \bullet\!| \, k \right\}.$$

For $\mathcal{S} := \{6, 7, 10\}$, the Inclusion-Exclusion formula (using the floor fnc) for the number of elements, $|\boldsymbol{\mu}_{\mathcal{S}}(N)|$, is

____

__ __ **c** In $[5x^2 + 4y + z^3 + 7]^{20}$, compute these coeffs:

$\text{Coeff}(x^6 z^8) =$ ________________ .

$\text{Coeff}(y^5 z^6) =$ ________________

[You may write answers as a product numbers, powers and multinomial-coeffs.]

__ __ **d** The physics lab has atomic *zinc*, *tin*, *silver* and *gold*. I'm allowed to take 6 atoms, so I have [expressed as single integer] ________________ many possibilities.

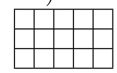
__ __ **e** The ***Chase-numbers*** comprise $\mathcal{C} := 1 + 3\mathbb{N}$.

$\mathcal{C}$-number $385 \overset{\text{note}}{=\!=} 35 \cdot 11$ is $\mathcal{C}$-*irreducible*: *T* *F*

Chase $N := 85$ is **not** $\mathcal{C}$-*prime* because $\mathcal{C}$-numbers $J :=$ ________ and $K :=$ ________ satisfy that $N \bullet\!| \, [J \cdot K]$, **yet** $N \nmid J$ and $N \nmid K$.

Also, $\mathcal{C}\text{-GCD}(175, 70) =$ ________ .

OYOP: *In grammatical English* ***sentences****, write your essay on every* 2^{nd} *line* (usually)*, so that I can easily write between the lines.*

U6: An ***Lmino*** (pron. "ell-mino") comprises three  squares in an "L" shape (all four orientations are allowed). For natnum N, let $\mathbf{R}_N$ denote the $3 \times N$ board: I.e,  is the $\mathbf{R}_5$ board. Prove:

Theorem: When N is odd, then board $\mathbf{R}_N$ is ***not*** *Lmino-tilable.*

You will likely want to first *state* and *prove* a Lemma. Now use appropriate induction on N to prove the thm. Also: *Illustrate your proof* with (probably several) large, *labeled* pictures.

Also, for $N = 2H$ *even*, our $\mathbf{R}_N$ has exactly ________ many Lmino-tilings (with proof).

U7: Consider a list $(n_1, n_2, n_3, \ldots, n_9)$ of integers. Use PHP [Pigeon-hole Principle] to prove there exists a *non-void* set $\Omega \subset [1..9]$ of indices, st.

$$\left[\sum_{j \in \Omega} n_j\right] \;|\!\bullet\; 9.$$

Use $\equiv$ for $\equiv_9$ i.e, congruence mod-9.

End of Class-U

U5:	__ __	90pts
U6:	__ __	50pts
U7:	__ __	35pts
Total:	__ __ __	175pts