

Hello. Use L_M for the lefthand-action of matrix M . Use B^t for the transpose of B . When working over \mathbb{Z}_p , state answers using *symmetric residues*, e.g. in \mathbb{Z}_{13} , answers should lie in $[-6..6]$.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\}$ $\neq 0$.

U1: Show no work.

y Prof. King wears bifocals, and cannot read small handwriting. **Circle** one: **True!** **Yes!** **Who??**

z Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, \$500 total] is a *really good idea*. A ten-page essay is fine. Date for the emailed-PDF is Sunday, March 27, 2022.

Circle: **Yes** **True** **Résumé material!**

a Shear the plane *vertically*, sending e_1 to $e_1 + 3e_2$, followed by the *horizontal* shear which sends e_2 to $-2e_1 + e_2$. Let S be the 2×2 matrix whose lefthand action is the preceding composition of shears.

Then $S = \left[\begin{array}{c|c} & \\ \hline & \end{array} \right]$.

b Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) := \begin{bmatrix} 3x - y \\ 2x + 6y \end{bmatrix}$. W.r.t ordered-basis $\mathcal{B} := \left(\begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \end{bmatrix}\right)$, let $M := \llbracket T \rrbracket_{\mathcal{B}}^{\mathcal{B}}$. Then $M = RTR^{-1}$,

where $R = \left[\begin{array}{c|c} & \\ \hline & \end{array} \right]$, $M = \left[\begin{array}{c|c} & \\ \hline & \end{array} \right]$.

c In each blank below, write either "there exist" or "for all", **Circle** one of the underlined scalar-pairs, and **Circle** a phrase.

Assertion $\text{Spn}(\mathbf{v}, \mathbf{w}) \supset \text{Spn}(\mathbf{x}, \mathbf{y})$ means:

"

 scalars a, b | c, d (st. | we have that | and)

 scalars a, b | c, d (st. | we have that)

 $a\mathbf{v} + b\mathbf{w} = c\mathbf{x} + d\mathbf{y}$."

d The 3×3 elem-matrix whose lefthand action adds

8 times row-2 to row-1 is $\left[\begin{array}{c|c|c} & & \\ \hline & & \\ \hline & & \end{array} \right]$.

U2: OYOP: Essay: *Write on every second line, so that I can easily write between the lines.*

i Consider a lin.trn $T: \mathbf{X} \rightarrow \mathbf{U}$ between finite-dimensional VSes. Distinguishing between zero-vectors $\mathbf{0}_X$ and $\mathbf{0}_U$, give, using set-builder notaion: A formal defn of $\text{Range}(T)$. And: A formal defn of $\text{Nul}(T)$.

ii Carefully state the Rank+Nullity theorem.

iii Give a careful proof of the Rank+Nullity thm for $T: \mathbf{X} \rightarrow \mathbf{U}$. Also: Use good, large *pictures* to illustrate the ideas in the proof.

End of Class-U

U1: 115pts

U2: 75pts

Total: 190pts

NAME:

HONOR CODE: "I have neither requested nor received help on this exam other than from my professor."

Signature:
