Differential Eqns MAP2302 **U-Class** Prof. JLF King Wedn, 03Mar2021

Welcome. Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$. **Write unambiguously e.g,** $1/a+b$ **should be *bracketed* either** $[1/a]+b$ **or** $1/[a+b]$**, as appropriate. (Be careful with negative signs!)**

Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ **or** $[\sin x]$**.**

U1: Show no work.

6 6 6 6 **a** For CCLDOp $\mathsf{L} := \mathbf{D}^3 - 3\mathbf{D} + 2\mathbf{I}$ and thrice diff'able fnc f, note $\mathsf{L}(f \cdot e^{2t}) = \mathsf{V}(f) \cdot e^{2t}$, where CCLDOp V is

$\mathsf{V} =$ ______ $\mathbf{D}^3 +$ ______ $\mathbf{D}^2 +$ ______ $\mathbf{D} +$ ______ $\mathbf{I}$.

[Put the correct number in *each* of the four blanks; zero, one, fractions, and negative numbers are allowed.]

10 20 10 **b** Bacteria with birth-multiplier $\mathbf{B}$ are in a petri dish with carrying capacity $\mathbf{C}$. The population, $p(t)$, satisfies the Logistic DE [write $p(t)$ rather than p, etc.] which is ______.

For *Hysteria* bacteria, $\mathbf{B} = \frac{1/5}{\text{min}}$. This petri dish has $\mathbf{C}=16\text{oz}$, with initial population $\mathbf{p}_0 = 2\text{oz}$. The time, $\boldsymbol{\tau}$, when *Hysteria* reaches half the carrying capacity is $\boldsymbol{\tau} = \widehat{\tau}\,\text{min}$, where $\widehat{\tau} =$ ______. [You may use exp() and log() to express your answer.]

Rounding $\widehat{\tau}$ up or down (your choice) to integer N, this $N =$ ______.

15 15 **c** DE $\left[[2x+8]y \cdot \frac{dy}{dx}\right] + 4y^2 = 0$ is not, alas, *exact*. Happily, multiplying both sides by (non-constant) function [a pure fnc of y alone]

$V(y) :=$ ______

gives a *new* DE which *is* exact.

Solving the exact-DE, every (non-zero) soln $y=y(x)$ satisfies $F\big(x, y(x)\big) = \alpha$, for some constant α, where

$F(x,y) :=$ ______.

6 6 6 6 **d** DE $h'' - 6h' + 13h = 0$, has fundamental-set of solutions $\{e^{\alpha t}, e^{\beta t}\}$, for [possibly complex] numbers $\alpha=$ ______ and $\beta=$ ______.

Alternatively, we can write our fundamental-set as

$$e^{Jt} \cdot \cos(Kt) \quad \text{and} \quad e^{Jt} \cdot \sin(Kt),$$

for *real* numbers $J=$ ______ and $K=$ ______.

10 15 **e** U.F. $y = y(t)$ satisfies

$$\dagger: \qquad y' + 3y^5 + t^2 y = 0.$$

Using a Bernoulli substitution of $z(t) := [y(t)]^K$, this becomes FOLDE $z' + [C(t) \cdot z] = G(t)$, where $K =$ ______ and $G(t) =$ ______.

7 **f** *"I have neither requested nor received help on this exam other than from my professor."*

U1: ___ ___ ___ 150pts

Total: ___ ___ ___ 150pts