Differential Eqns MAP2302 | **U-Class** | Prof. JLF King Wedn, 03Mar2021

U1: Show no work.

6 6 6 6 **a** For CCLDOp $\mathsf{L} := \mathbf{D}^3 - 3\mathbf{D} + 2\mathbf{I}$ and thrice diff'able fnc f, note $\mathsf{L}(f\cdot e^{2t}) = \mathsf{V}(f)\cdot e^{2t}$, where CCLDOp V is

$\mathsf{V} =$ ______ $\mathbf{D}^3 +$ ______ $\mathbf{D}^2 +$ ______ $\mathbf{D} +$ ______ $\mathbf{I}$.

[Put the correct number in *each* of the four blanks; zero, one, fractions, and negative numbers are allowed.]

10 20 10 **b** Bacteria with birth-multiplier $\mathbf{B}$ are in a petri dish with carrying capacity $\mathbf{C}$. The population, $p(t)$, satisfies the Logistic DE [write $p(t)$ rather than p, etc.] which is ______________________.

For *Hysteria* bacteria, $\mathbf{B} = \frac{1/5}{\text{min}}$. This petri dish has $\mathbf{C}$=16oz, with initial population $\mathbf{p}_0 = 2$oz. The time when *Hysteria* has reached half the carrying capacity is τ min, where $\tau =$ ______________. [You may use exp() and log() to express your answer.]

Rounding τ up or down (your choice) to integer N, this $N =$ ______.

15 15 **c** DE $[[2x+8]y \cdot \frac{dy}{dx}] + 4y^2 = 0$ is <u>not</u>, alas, *exact*. Happily, multiplying both sides by (non-constant) fnc

$V(y) =$ ______________________

gives a *new* DE which <u>*is*</u> exact.

Solving the exact-DE, every (non-zero) soln $y=y(x)$ satisfies $F\big(x, y(x)\big) = \alpha$, for some constant α, where

$F(x,y) =$ ______________________.

6 6 6 6 **d** DE $h'' - 6h' + 13h = 0$, has fundamental-set of solutions $\{e^{\alpha t}, e^{\beta t}\}$, for [possibly complex] numbers $\alpha=$ __________ and $\beta=$ __________.

Alternatively, we can write our fundamental-set as

$$e^{Jt}\cdot\cos(Kt) \quad \text{and} \quad e^{Jt}\cdot\sin(Kt),$$

for *real* numbers $J=$ ______ and $K=$ ______.

10 15 **e** U.F. $y = y(t)$ satisfies

$$\dagger: \quad y' + 3y^5 + t^2 y = 0.$$

Using a Bernoulli substitution of $z(t) := [y(t)]^K$, this becomes FOLDE $z' + [C(t)\cdot z] = G(t)$, where $K =$ ______ and $G(t) =$ ______________.

7 **f** *"I have neither requested nor received help on this exam other than from my professor."*

U1: ___ ___ ___ 150pts

Total: ___ ___ ___ 150pts