Staple!

Ord: ___ ___

Differential Eqns MAP2302 — **U-Class** — Prof. JLF King, Wedn., 03Apr2019

Hi. Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0 \neq$ *Empty-word*. **Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible** $\sin x$ **or** $[\sin x]$**.**

U1: Show no work.

___ ___ a) The visual representation of $\mathbb{C}$ is sometimes called "the ? plane", where ? is Circle: **Unreal** **Higher** **Snakes-on-a** **Argand** **Krypton** **Radon** **Xenon** **Euler** **Gauss** **Please-x** **y-com** **Air** **Sea** **De** **Rain-in-Spain-stays-mainly-on-the** .

___ ___ b) A multivariate polynomial, where each monomial has the same degree, is Circle: **level** **uniform** **monogamous** **delicious** **flat** **polyandrous** **manic** **unitary** **Unitarian** **utilitarian** **monic** **smooth** **penultimate** **homogeneous**

C Inverse of $\mathsf{C} := \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$, is $\mathsf{C}^{-1} =$ ______________.

Conjugating $\mathsf{W} := \begin{bmatrix} 9 & -5 \\ 10 & -6 \end{bmatrix}$ by C

gives diagonal matrix $\mathsf{D} := \mathsf{C}^{-1}\mathsf{W}\mathsf{C} =$ ______________.

Thus the $(2,1)$-entry of $\mathsf{e}^{t\mathsf{W}}$ is ____________________.

___ d Consider linear DiffOp

$$\mathsf{V}(y) \;:=\; ty'' - [1+t]y' + y\,.$$

Verify [for yourself] that $\mathsf{V}(Y_0) = 0$ and $\mathsf{V}(Y_1) = 0$, where $Y_0 := \mathsf{e}^t$ and $Y_1 := 1+t$. Their Wronskian is $\mathcal{W}(Y_0, Y_1) =$ .... Then VoP tells us that

$y_{\alpha,\beta} :=$ ....

is the general soln to $\mathsf{V}(y_{\alpha,\beta}) = 3t^2$.

WAlph: [t * y''(t)] - [1+t]y'(t) + y(t) = 3 t^2

Interestingly, WlfAlph does *not* detect the full simplification. Here, humans go one step better...

U2: Show no work.

e $\mathbb{R}$-matrices $\mathsf{V}, \mathsf{H}, \mathsf{A}, \mathsf{B}$ are 3×3, with V invertible and A, B each nilpotent. [Use $\mathbf{I}$ for the 3×3 id-matrix.]

Matrix $\mathsf{e}^{[\mathsf{H}+\mathbf{I}]\mathsf{H}}$ *equals* $\mathsf{e}^{\mathsf{H}} \cdot \mathsf{e}^{\mathsf{H}^2}$:	*AT*	*AF*	*Nei*
Matrix $\mathsf{e}^{[\mathsf{H}^2]}$ *equals* $[\mathsf{e}^{\mathsf{H}}]^2$:	*AT*	*AF*	*Nei*
A^2 *is the zero-matrix*:	*AT*	*AF*	*Nei*
Each entry of $\mathsf{e}^{t\mathsf{A}}$ *is a polynomial*:	*AT*	*AF*	*Nei*
Matrix e^{A} *is nilpotent*:	*AT*	*AF*	*Nei*
Matrix VA *is nilpotent*:	*AT*	*AF*	*Nei*
Matrix BA *is nilpotent*:	*AT*	*AF*	*Nei*
Matrix VAV^{-1} *is nilpotent*:	*AT*	*AF*	*Nei*

f U.F. $x = x(t)$ satisfies $\boxed{2x^{(3)} + 5x^{(2)} - x \;=\; 0}$.

Then $\mathsf{Y} := \begin{bmatrix} x \\ x' \\ x'' \end{bmatrix}$ satisfies $\mathsf{Y}' = \mathsf{M}{\cdot}\mathsf{Y}$, where M is this 3×3 matrix of numbers:

$$\mathsf{M} \quad = \quad \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}.$$

g Fncs $x(t)$ and $y(t)$ satisfy this system of DEs,

$$\begin{aligned} x' + x - 3y &= 0\,, \\ y' + 6x - 8y &= 0\,. \end{aligned}$$

It can be written as $\mathsf{Y}' = \mathsf{M}\cdot\mathsf{Y}$,
where $\mathsf{Y} := \left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$ and M is matrix ________________.

Characteristic poly of M is $\wp_{\mathsf{M}}(z)=$ ________________

A soln has $x(t)$ a linear combination of $e^{\alpha t}$ and $e^{\beta t}$ for *numbers* $\alpha=$ ____________ and $\beta=$ ____________.

End of U-Class

U1:	___ ___ ___	130pts
U2:	___ ___ ___	150pts
Total:	___ ___ ___	280pts