Staple!

Sets and Logic
MHF3202 2787

Class-U

Prof. JLF King
31Aug2015

U1: Short answer. Show no work.

Please write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

__ __ a) Define $G{:}[1..12] \circlearrowleft$ where $G(n)$ is the number of letters in the n^{th} Gregorian month. So $G(2) = 8$, since the 2^{nd} month is "February". The only fixed-point of G is ______. The *set* of posints k where $G^{\circ k}(12) = G^{\circ k}(7)$ is ____________________.

__ __ b) Write the truth-table for $B \Rightarrow [[\neg B] \Rightarrow C]$.

OYOP: *In grammatical English* ***sentences****, write your essay on every* ***third*** *line* (usually), *so that I can easily write between the lines. Do* **<u>not</u>** *restate the question.*

U2: Define a sequence $\vec{\mathbf{b}} = (b_0, b_1, b_2, \ldots)$ by $b_0 := 0$ and $b_1 := 3$ and

$$\dagger: \quad b_{n+2} := 7b_{n+1} - 10b_n\,, \quad \text{for } n = 0, 1, \ldots.$$

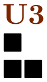
Use induction to prove, for all $k \in [0..\infty)$, that

$$\ddagger: \quad b_k = 5^k - 2^k\,.$$

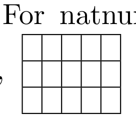
Further. Given recurrence (†) and initial conditions, *explain* how you could have discovered/computed the numbers 5 and 2 in the (‡) formula.

Can you generalize to getting a (‡)-like formula when the recurrence is $b_{n+2} := \mathbf{S}b_{n+1} - \mathbf{P}b_n$, for arbitrary real-number coefficients $\mathbf{S}$ and $\mathbf{P}$?

U3: An ***Lmino*** (pron. "ell-mino") comprises three



squares in an "L" shape (all four orientations are allowed). For natnum N, let $\mathbf{B}_N$ denote the $3{\times}N$ board: I.e,



is the $\mathbf{B}_5$ board. Prove:

Theorem: When N is odd, then board $\mathbf{B}_N$ is <u>not</u> Lmino-tilable.

You will likely want to first *state* and *prove* a Lemma. Now use appropriate induction on N to prove the thm. Also: *Illustrate your proof* with (probably several) large, *<u>labeled</u>* pictures.

U1: ___ ___ 40pts

U2: ___ ___ 90pts

U3: ___ ___ 60pts

Total: ___ ___ ___ 190pts

Please PRINT your name and ordinal. Ta:

Ord: ____________________

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ____________________