NT&Cryp MAT4930 **Prereq-U** Prof. JLF King Friday, 08Jan2016

U1: ___ ___ ___ 105pts

Total: ___ ___ ___ 105pts

Show no work. *NOTE*: The **inverse-fnc** of g, often written as g^{-1}, is *different* from the **reciprocal fnc** $1/g$. E.g, suppose g is invertible with $g(-2) = 3$ and $g(3) = 8$: Then $g^{-1}(3) = -2$, yet $[1/g](3) \stackrel{\text{def}}{=} 1/g(3) = 1/8$.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

Please PRINT your Name

....

Please PRINT your Student-ID number (just this once)

....

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....

U1:

___ ___ a) On $\mathbb{Z}_+$, write $x \ \$ \ y$ IFF $xy < 0$. So \$ is Circle

Transitive T F. **Symmetric** T F. **Reflexive** T F.

On $\mathbb{Z}$, say that $x \nabla y$ IFF $x - y \leqslant 1$. Then ∇ is:

Trans. T F. **Symm.** T F. **Reflex.** T F.

(Be *careful* on both parts!)

___ ___ b) Let $\mathcal{P}_\infty$ denote the family of all *infinite* subsets of $\mathbb{N}$. Define relation $\approx$ on $\mathcal{P}_\infty$ by: $A \approx B$ IFF $A \cap B$ is infinite. Stmt "*This $\approx$ is an equivalence-relation*" is: T F

___ ___ c) $\left[\sqrt{2}^{\sqrt{27}}\right]^{\sqrt{3}} =$ .... $\log_8(4) =$ ....

___ ___ d) Quadratic $15x^2 + 23x + 6 = [Ax - \alpha] \cdot [Bx - \beta]$, for numbers $A=$ ...... , $\alpha=$ ...... ; $B=$ ...... , $\beta=$ ....

___ ___ e) Below, f and g are differentiable fncs with

$$\begin{array}{llll} f(2) = 3, & f(3) = 5, & f'(2) = 19, & f'(3) = 17, \\ g(2) = 11, & g(3) = 13, & g'(2) = \frac{1}{2}, & g'(3) = 7, \\ f(5) = 43, & g(5) = 23, & f'(5) = 41, & g'(5) = 29\,. \end{array}$$

Define the composition $C \coloneqq g \circ f$. Then $C(2) =$ .... ; $C'(2) =$ ....

Please write each answer as a product of numbers; **do not** multiply out. [*Hint:* The Chain rule.]

___ ___ f) Let $y = f(x) \coloneqq \left[7 + \sqrt[3]{2x}\right]/5$. Its inverse-function is $f^{-1}(y) =$ ....

___ ___ g) Compute the sum of this geometric series: $\sum_{k=1}^{\infty} [-1]^k \cdot [1/3]^{2k} =$ ....

End of Prereq-U