Entropy, Meshalkin, and Determinism

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ABSTRACT: A little part of the talk I gave, at Matt Foreman's invitation, at UC Irvine, on Thursday, 23May2002.

Meshalkin's example

Let $\mathbb{X}$ be the, doubly-infinite, shift-space over ***"digit* alphabet"** $\{0, +1, -1, +2, -2\}$. Let $S{:}\mathbb{X}\circlearrowleft$ be the independent $(\frac{1}{2}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8})$-process on $\mathbb{X}$.

And let $T{:}\mathbb{Y}\circlearrowleft$ be the independent $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$-process over ***"letter* alphabet"** $\{\mathtt{E}, \mathtt{D}, \mathtt{P}, \mathtt{N}\}$; Even, oDd, Positive, Negative.

Each of $+1, -1, +2, -2$ is a ***NZD***; a Non-Zero Digit. Given a point x

$$\begin{array}{ccccccccccccc} \dots & \mathbf{0} & \mathbf{0} & \mathbf{0} & -1 & \mathbf{0} & \mathbf{0} & +1 & +2 & -1 & +1 & \mathbf{0} & \dots \\ \dots & (& (& (&) & (& (&) &) &) &) & (& \dots \end{array}$$

regard each $\mathbf{0}$ as a left-parenthesis, and regard each NZD as a right-parenthesis; this is shown above, in the lower line. I.e, for each pair of the form 0 NZD, [i.e $x_n = 0$ and x_{n+1} is an NZD] link up the pair, then mentally "delete" these pairs. Apply this idea recursively.

$$\begin{array}{ccccccccccc} \mathbf{0} & \mathbf{0} & \mathbf{0} & -1 & \mathbf{0} & \mathbf{0} & +1 & +2 & -1 & +1 & \mathbf{0} \\ \mathtt{P} & \mathtt{N} & \mathtt{N} & \mathtt{D} & \mathtt{P} & \mathtt{P} & \mathtt{D} & \mathtt{E} & \mathtt{D} & \mathtt{D} & ? \end{array}$$

Below each $\mathbf{0}$, write "P" or "N" as the $\mathbf{0}$ is linked to a *positive* or *negative* digit. And below the other digits, write "E" or "D" as the digit is *even* or *odd*. So the upper string is mapped to the lower strip.

Cylinder sets. A (*basic*) ***cylinder set*** is specified by listing symbols at an interval of coordinates. E.g

$$\mathbf{C} := [\![+2\ +2\ -1\ \mathbf{0}\ \mathbf{0}\ +1]\!]_7$$

is the set of $x \in \mathbb{X}$ st.

$$x_7 = +2,\ x_8 = +2,\ x_9 = -1,\ x_{10} = \mathbf{0},\ x_{11} = \mathbf{0},\ x_{12} = +1\,.$$

We define the probability, $\mathsf{Pr}(\mathbf{C})$, to be the product of the probabilities of the individual symbols. Here,

$$\mathsf{Pr}(\mathbf{C}) \quad = \quad \tfrac{1}{8} \cdot \tfrac{1}{8} \cdot \tfrac{1}{8} \cdot \tfrac{1}{2} \cdot \tfrac{1}{2} \cdot \tfrac{1}{8}\,.$$

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