Differential Eqns MAP2302 3228 **S-Class** Prof. JLF King Wedn., 26Sep2018

S1: Show no work.

b DE $\left[xe^y \cdot \frac{dy}{dx}\right] + \left[8x^4 + 4e^y\right] = 0$ is not, alas, *exact*. Happily, multiplying both sides by (non-constant) fnc

$W(x) =$ ______________________

gives a *new* DE which *is* exact. **Did you** *Check?*

Exactifying soln: Note $\mathcal{N}_x - \mathcal{M}_y = e^y - 4e^y$. Thus

$$\frac{\mathcal{N}_x - \mathcal{M}_y}{\mathcal{N}} = \frac{-3 \cdot e^y}{x \cdot e^y} = \frac{-3}{x}, \quad \textit{a pure fnc of } x.$$

Our coeff-fnc is $C(x) := \frac{-3}{x}$, with antideriv $B := -3\log$. Hence mult-fnc is $M(x) := e^{-3\log(x)} = 1/x^3$. So our exactifier is $W(x) := \frac{1}{M(x)} = x^3$.

c For $t{>}0$, fnc $y_\alpha(t) :=$ ______________________ is the gen.soln to $ty' + 3y = t^7$. [*Hint:* FOLDE.]

Soln: Using our FOLDE notation, the coeff and tar fncs are $C(t) := \frac{3}{t}$ and $G(t) := t^6$. Whence $B := \int C$ is

$$B(t) = 3\log(t). \quad \textit{Hence, } M(t) := e^{3\log(t)} = t^3.$$

So $P(t) := t^3 \cdot G(t) = t^9$ and $Q(t) := \int^t P = \frac{1}{10}t^{10}$. Thus

$$y_\alpha(t) = \frac{\alpha + Q(t)}{M(t)} \overset{\text{note}}{=\!=} \frac{\alpha}{t^3} + \frac{1}{10}t^7$$

is our general soln to the given DE. [Did you check?]

d Let $U := 3 - 2\boldsymbol{i}$ and $W := 4 + \boldsymbol{i}$. The gen.soln to a CCLDE is $\boxed{y_{\alpha,\beta}(t) = \alpha \cdot e^{Ut} + \beta \cdot e^{Wt}}$. The CCLDE that every such $y()$ satisfies is

______________________ $= 0$.

[*Hint:* Fill-in the blank with the appropriate sum of derivatives-of-y times various constants.]

Soln: Jeopardy DE. The desired aux-poly is

$$q(z) := [z - U].[z - W] \overset{\text{note}}{=\!=} z^2 - [U + W]z + [U \cdot W].$$

We now realize we need to compute the sum and product of the two given roots. Easily, $U{+}W = [7 - \boldsymbol{i}]$ and $U \cdot W = [14 - 5\boldsymbol{i}]$. Consequently, our sought-after DE is

$$y'' - [7 - \boldsymbol{i}] \cdot y' + [14 - 5\boldsymbol{i}] \cdot y = 0.$$

e The simplest soln to $y'' + 2y' + y = [t^2 + 1]/e^t$ is $y(t) =$ ______________________.

PolyExp: For $\mathsf{L} := \mathbf{D}^2 + 2\mathbf{D} + \mathbf{I}$, fnc f, and $E := e^{-t}$, note $[fE]' = [f' - f]E$ and $[fE]'' = [f'' - 2f' + f]E$. So $\mathsf{L}(fE) = f'' \cdot E$.

Our target $G := t^2 + 1$ has degree 2, so f has form $wt^4 + vt^3 + ut^2$. Hence $f'' = 12wt^2 + 6vt + 2u$. Equality $f'' = G$ says $w = \frac{1}{12}$ and $v = 0$ and $u = \frac{1}{2}$.

Thus $y = \left[\frac{t^4}{12} + \frac{t^2}{2}\right] \cdot e^{-t}$. As usual, the general solution adds $\alpha \cdot e^{-t} + \beta \cdot te^{-t}$.

f DiffOperators $\mathbf{P}, \mathbf{Q}, \mathbf{R}, \mathbf{S}$ are defined as

$$\mathbf{P}(f) := f(3) \cdot f', \qquad \mathbf{Q}(f) := \cos(3) \cdot f^{(3)},$$
$$\mathbf{R}(f) := [\cos(3) \cdot f] + f'', \qquad \mathbf{S}(f) := \cos(3) + [3f'].$$

Then... **P** *is linear:* T (F). **Q** *is linear:* (T) F.
R *is linear:* (T) F. **S** *is linear:* T (F).

S2: Show no work.

g Degree-N polynomial $y = y(t)$ satisfies

$$\dagger: \quad 4y^2 - t^9 y' = 15t^9 + 4t^2.$$

Thus $N =$ 8 . [*Hint:* Don't compute y; just the polynomial's degree.]

Soln: As polynomials, let

$$\mathcal{E} := \operatorname{Deg}(4y^2) \quad \text{and} \quad \mathcal{B} := \operatorname{Deg}(t^9 y').$$

If $\boxed{\mathcal{E} \neq \mathcal{B}}$, then $\operatorname{Max}(\mathcal{E}, \mathcal{B}) \overset{\text{must}}{=\!=} \operatorname{Deg}(\operatorname{RhS}(\dagger)) = 9$. But $\mathcal{E}$ is even, thus $\mathcal{B} = 9$. So $y' = -15$, whence $y = -15t + K$, for some number K. Hence the t^2-coefficient of LhS(†) is $4 \cdot [-15]^2$, which does *not* equal 4, the t^2-coeff of RhS(†).

Thus $\boxed{\mathcal{E} = \mathcal{B}}$, so $2N = 9 + [N-1]$. [*DE (†) has no const-solns, hence* $\operatorname{Deg}(y')$ *really is* $N-1$.] So $N = 8$.

[Aside: Polynomial $y(t) := t + 2t^8$ satisfies (†).]

[Aside: For $u = u(t)$, define operator $\mathsf{E}(u) := 4u^2 - t^9 u'$. A zeroTar version of (†) is $\boxed{\mathsf{E}(u) = 0}$. This DE can be written as $\frac{du}{u^2} = \frac{4\,dt}{t^9}$; a separable DE. Integrating gives

$$\frac{1}{-1} \cdot u^{-1} = -\alpha + \frac{1}{-2} \cdot t^{-8} = -\alpha - \frac{1}{2\,t^8} = -\frac{2\alpha t^8 + 1}{2\,t^8}. \quad \text{So,}$$

$$\ddagger: \quad u_\alpha(t) = \frac{2\,t^8}{2\alpha t^8 + 1}.$$

CAVEAT: Must $\big[[t + 2t^8] + u_\alpha\big]$ be a soln to (†)? ***No!*** For note that operator $\mathsf{E}()$ is *not* linear, due to the u^2 term.

BTWay, note that DE $4u^2 - t^9 u' = 0$, is a Bernoulli DE, when written $u' = \frac{4}{t^9} u^{-[-1-1]}$. With a CoV it becomes a FOLDE.]

S1:	___ ___ ___	120pts
S2:	___ ___	30pts
Total:	___ ___ ___	150pts