Combinatorics 1
MAD4203

Sample-S

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Welcome: This is a practice prereq exam. We'll have the actual prereq exam (which will be shorter)*, in class, on* **Friday, 24Aug**. *Please bring paper for computations.*

The prereq counts only 4%–8% of the course grade. It's purpose is for you to self-evaluate if you have the prerequisite knowledge for the COMBINATORICS 1 *course. If you've forgotten a little, that's fine, but if you are shaky on* **a lot** *of material, then you should consider using this semester to review, and taking the course in Spring.*

S1: Show no work. *NOTE*: The **inverse-fnc** of g, often written as g^{-1}, is *different* from the **reciprocal fnc** $1/g$. E.g, suppose g is invertible with $g(-2) = 3$ and $g(3) = 8$: Then $g^{-1}(3) = -2$, yet $[1/g](3) \stackrel{\text{def}}{=\!=} 1/g(3) = 1/8$.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

ℓ1 The **slope** of line $3[y-5] = 2[x-2]$ is ________.
Point $(-4, y)$ lies on this line, where y= ________.

ℓ2 Line $y = [M \cdot x] + B$ owns points $(4,3)$ and $(-2,5)$. Hence M= ________ and B= ________.

ℓ3 Line $y = Mx + B$ is orthogonal to $y = \frac{1}{3}x + 2$ and owns $(2,1)$. So M= ________ and B= ________.

q1 The solutions to $3x^2 = 2 - 2x$
are $x =$ ________.

q2 The four solutions to $[y-2] \cdot y \cdot [y+2] = -1/y$
are $y =$ ________

[*Hint:* Apply the Quadratic Formula to y^2.]

e1 $\left[\sqrt{3}^{\sqrt{2}}\right]^{\sqrt{8}}$= ________. $\log_{64}(16)$= ________.

iv1 Let $y = f(x) := \left[2 + \sqrt[5]{x}\right]/3$. Its inverse-function is $f^{-1}(y)$= ________.

iv2 Suppose g is a fnc with g' never zero. Let h be the inverse-fnc of g. In terms of h, g, g' and x, write a formula for $h'(x)$= ________.

[*Hint:* The Chain rule. NOTE: h is **NOT** $1/g$.]

iv3 Let $g(x) := x^3 - x$. Then $g^{-1}(6)$= ________
and $[g^{-1}]'(6)$= ________.

dq $\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\sin(3z)}{\cos(z+1)}\right) = \frac{f(z)}{g(z)}$ where
$f(z)$= ________
and $g(z)$= ________.

DC1 Below, f and g are differentiable fncs with

$$f(2) = 3,\quad f(3) = 5,\quad f'(2) = 19,\quad f'(3) = 17,$$
$$g(2) = 11,\quad g(3) = 13,\quad g'(2) = \tfrac{1}{2},\quad g'(3) = 7,$$
$$f(5) = 43,\quad g(5) = 23,\quad f'(5) = 41,\quad g'(5) = 29\,.$$

Define the composition $C := g \circ f$. Then
$C(2) =$ ________ ; $C'(2) =$ ________.

Please write each answer as a product of numbers; **do not** multiply out. [*Hint:* The Chain rule.]

DC2 For $x > 0$, let $B(x) := x^x$. Its derivative is
$B'(x)$= ________.

[*Hint:* How is y^z, for $y > 0$, defined in terms of the exponential fnc?]

DC3 For $x > 0$, let $B(x) := x^{\sin(x)}$. Hence its derivative is $B'(x) = B(x) \cdot M(x)$, where $M(x)$ equals
________.

[*Hint:* How is y^z, for $y > 0$, defined ITOf the exponential fnc?]

DC4 On those x where $\sin(x) > 0$, define $B(x) := [\sin(x)]^x$. Its derivative is
$B'(x)$= ________.

[*Hint:* How is y^z, for $y > 0$, defined ITOf the exponential fnc?]

Ig1 $\int_2^3 \log(t)\,\mathrm{d}t = \log(R) + K$, where R= ________
is a rational number and K= ________ $\in \mathbb{Z}$. [*Hint:* IBParts]

Ig2 $\int \frac{t^2}{2^t}\,\mathrm{d}t =$ ________. [Write ITOf L:=log(2).]

Hop By l'Hôpital's thm or other means, please compute
$\lim_{x \to 0} \frac{\mathrm{e}^x - 1}{3x - 1}$= ________. $\lim_{x \to \frac{\pi}{2}} \frac{\sin(2x)}{x - \frac{\pi}{2}}$= ________.

Paf Partial-fraction decomposition:

$$\frac{x+1}{x^2+x-2} = \underline{\qquad\qquad} + \underline{\qquad\qquad}.$$

Sm Sum $1+2+3+\ldots+100=$ ____________.

Tel1 Sum $\sum_{k=2}^{\infty} \frac{1}{k[k+2]} = \frac{U}{D}$, where $U=$ ________ and $D=$ ________________ are co-prime posints.

[*Hint:* Telescoping series (several). "Co-prime" means "with no common factor".]

Tel2 Let $b_n := \exp(\frac{4}{n+5}) \stackrel{\text{note}}{=\!=} \mathrm{e}^{4/[n+5]}$. Then

$\sum_{n=4}^{\infty} [b_n - b_{n+1}] =$ ________________________.

[*Hint:* A real, $+\infty$, $-\infty$ or **DNEin**$\overline{\mathbb{R}}$.]

SG1 Compute the sum of this geometric series: $\sum_{n=3}^{\infty} [-1]^n \cdot [3/5]^n =$ ________________________.

SG2 For natural number K, the sum $\sum_{n=3}^{3+K} 4^n$ equals ________________________.

SG3 $\sum_{n=1}^{\infty} r^n = \frac{5}{8}$. So $r=$ ____________ or **DNE**.

[*Hint:* The sum starts with n at **one**, not zero.]

SC The series $\sum_{k=1}^{\infty} \frac{[-1]^k}{\log(k)}$ (circle one): Diverges, Converges absolutely, Converges conditionally.

SR Power series $f(x) = \sum_{n=3}^{\infty} \frac{[3x]^n}{n+7}$ has RoC= ______.

S2: Math-Greek alphabet: Please write the **two** missing data of lowercase/uppercase/name. Eg:

"iota: ___ α: ___ B: ___." You fill in: ι I A *alpha* β *beta*.

Ω: ________ Ψ: ________ H: ________

σ: ________ γ: ________ λ: ________

theta ______ rho ______ delta ______ mu ______

End of Sample-S

S1:	___ ___ ___	180pts
S2:	___ ___	20pts
Total:	___ ___ ___	200pts

Print name ________________________________ Ord:

Honor Code: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ________________________________