Number Sets. Expression $k \in \mathbb{N}$ [read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$"] means that k is a natural number; a ***natnum***. Expression $\mathbb{N} \ni k$ [read as "$\mathbb{N}$ ***owns*** k"] is a synonym for $k \in \mathbb{N}$.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \dots\}$.

$\mathbb{Z}$ = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive rationals and $\mathbb{Q}_-$ for the negative rationals.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

For $\omega \in \mathbb{C}$, let "$\omega > 5$" mean "ω ***is real*** and $\omega > 5$". [Use the same convention for $\geqslant, <, \leqslant$, and also if 5 is replaced by any real number.]

An "***interval of integers***" $[b\,..\,c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e}\,..\,2\pi] = \{3, 4, 5, 6\} = [3\,..\,6] = (2\,..\,6]$. We allow b and c to be $\pm\infty$; so $(-\infty\,..\,-1]$ is $\mathbb{Z}_-$. And $[-\infty\,..\,-1]$, is $\{-\infty\} \cup \mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2i| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'. RoC: '*Radius of Convergence*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand Side*' of an eqn or inequality. LhS: '*left-hand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', but pt: '*point*', as in "a fixed-pt of a map".

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** fnc, defined for $x>0$, is $\log(x) := \int_1^x \frac{\mathrm{d}v}{v}$. Its inverse-fnc is $\exp()$. For $x>0$, then, $\exp(\log(x)) = x = \mathrm{e}^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(\mathrm{e}^t)$.

PolyExp: '*Polynomial-times-exponential*'; e.g, $[3 + t^2] \cdot \mathrm{e}^{4t}$. PolyExp-sum: '*Sum of polyexps*'. E.g, $f(t) := 3t\mathrm{e}^{2t} + [t^2] \cdot \mathrm{e}^t$ is a polyexp-sum.

Phrases. WLOG: '*Without loss of generality*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. Use iff: '*if and only if*'.

IST: '*It Suffices to*' as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. inter alia: '*among other things*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Bi/Multi-nomial coeffs. For a natnum n, use "$n!$" to mean "n ***factorial***"; the product of all posints $\leqslant n$. So $3! = 3 \cdot 2 \cdot 1 = 6$ and $5! = 120$. Also $0! = 1 = 1!$.

For natnum K and arb. complex number β, define

Rising Fctrl: $[\![\beta \uparrow K]\!] := \beta \cdot [\beta+1] \cdot [\beta+2] \cdots [\beta+[K-1]]$,
Falling Fctrl: $[\![\beta \downarrow K]\!] := \beta \cdot [\beta-1] \cdot [\beta-2] \cdots [\beta-[K-1]]$.

E.g, $[\![K \downarrow K]\!] = K! = [\![1 \uparrow K]\!]$. Two further examples,

$$\left[\!\!\left[\tfrac{2}{7} \downarrow 4\right]\!\!\right] = \frac{2}{7} \cdot \frac{-5}{7} \cdot \frac{-12}{7} \cdot \frac{-19}{7} \text{ and } [\![1 \downarrow 3]\!] = 1 \cdot 0 \cdot -1 = 0\,.$$

In particular, for $n \in \mathbb{N}$: If $K > n$ then $[\![n \downarrow K]\!] = 0$. We pronouce $[\![5 \downarrow K]\!]$ as "5 falling-factorial K".

The ***binomial coefficient*** $\binom{7}{3}$, read "7 choose 3", means *the number of ways of choosing* 3 *objects from* 7 *distinguishable objects*. If we think of putting these objects in our left pocket, and putting the remaining 4 objects in our right pocket, then we write the coefficient as $\binom{7}{3,4}$. [Read as "7 choose 3-comma-4."] Evidently

$$\binom{N}{j} \xlongequal{\text{with } k := N-j} \binom{N}{j,\ k} = \frac{N!}{j!\,k!} = \frac{[\![N \downarrow j]\!]}{j!}\,.$$

Note $\binom{7}{0} = \binom{7}{0,7} = 1$. Also, $\binom{N+1}{k+1} = \binom{N}{k} + \binom{N}{k+1}$. Finally, the Binomial theorem says

$$B_1\textbf{:} \qquad [x+y]^N = \sum_{j+k=N} \binom{N}{j,k} \cdot x^j y^k\,,$$

where (j,k) ranges over all *ordered* pairs of natural numbers with sum N.

In general, for natnums $N = k_1 + \ldots + k_P$, the ***multinomial coefficient*** $\binom{N}{k_1,\, k_2,\, \ldots,\, k_P}$ is the number of ways of partitioning N objects, by putting k_1 objects in pocket-one, k_2 objects in pocket-two, …putting k_P objects in the P^{th} pocket. Easily

$$B_2\textbf{:} \quad \binom{N}{k_1, k_2, \ldots, k_P} = \frac{N!}{k_1! \cdot k_2! \cdot \ldots \cdot k_P!}\,.$$

And $[x_1 + \cdots + x_P]^N$ indeed equals the sum of terms

$$B_1'\textbf{:} \qquad \binom{N}{k_1,\ldots,k_P} \cdot {x_1}^{k_1} \cdot {x_2}^{k_2} \cdots {x_P}^{k_P}\,,$$

taken over all natnum-tuples $\vec{k} = (k_1, \ldots, k_P)$ that sum to N.

Operations on Sets. Use $\in$ for "is an element of". E.g, letting $\mathbb{P}$ be the set of primes, then, $5 \in \mathbb{P}$ yet $6 \notin \mathbb{P}$. Changing the emphasis, $\mathbb{P} \ni 5$ ["$\mathbb{P}$ *owns* 5"] yet $\mathbb{P} \not\ni 6$ ["$\mathbb{P}$ *does-not-own* 6"]

For subsets A and B of the same space, Ω, the ***inclusion relation*** $A \subset B$ means:

$$\forall \omega \in A, \text{ necessarily } B \ni \omega.$$

And this can be written $B \supset A$. Use $A \subsetneqq B$ for *proper* inclusion, i.e, $A \subset B$ yet $A \neq B$.

The *difference set* $B \smallsetminus A$ is $\{\omega \in B \mid \omega \notin A\}$. Employ A^c for the ***complement*** $\Omega \smallsetminus A$. Use $A \triangle B$ for ***symmetric difference*** $[A \smallsetminus B] \cup [B \smallsetminus A]$. Furthermore

$A \,\blacksquare\, B$,	Sets A & B have *at least* <u>one</u> point in common; they intersect.
$A \sqcap B$,	The sets have *no* common point; disjoint.

The symbol "$A \,\blacksquare\, B$" both asserts intersection <u>and</u> represents the set $A \cap B$. For a collection $\mathcal{C} = \{E_j\}_j$ of sets in Ω, let the ***disjoint union*** $\bigsqcup_j E_j$ or $\bigsqcup(\mathcal{C})$ represent the union $\bigcup_j E_j$ *and also* asserts that the sets are pairwise disjoint.

See next page…

SeLo [2020g] quizzes

P1: Fri. 24 Jan Multinomial coefficient $\binom{9}{4,2,3}$ = ________ = ________.

[*Note:* Write your ans. ITOf factorials, then **also** write it as a single integer, or product of two, **without** factorials.]

P2: Fri. 31 Jan Write the free vars in each of these expressions.

$$\overbrace{\exists n\in\mathbb{N}\colon\ f(n)\subset \underbrace{\bigcup_{\ell=p-4}^{p+7}\underbrace{\{x\in\mathbb{Z}\mid \ell\cdot n \equiv_5 x^2\}}_{E1}}_{E2}}^{E3}$$

$E3$: ________. $E2$: ________. $E1$: ________.

P3: Mon. 03 Feb The number of permutations of "PREPPER", as a multinomial coefficient, is ________ as a number ________.

P4: Wed. 19 Feb Relation $\mathbf{R}$ is a binrel on set $\mathbb{N}$, defined by $x\mathbf{R}y$ IFF $x^2 = 5y$.

Assertion "*Relation* $\mathbf{R}$ *is reflexive*" is *T* *F*

Assertion "*Relation* $\mathbf{R}$ *is antireflexive*" is *T* *F*

P5: Fri. 21 Feb Consider binrels on $\Omega :=$ Stooges $:= \{M, L, C\}$. There are ________ **Anti-reflexive** binrels, and ________ **Reflexive** binrels, and ________ **Symmetric** binrels. The number of **strict total-orders** is ________.

Due to COVID-19, *we had only 5 quizzes.*