Number Sets. An expression such as $k \in \mathbb{N}$ (read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$") means that k is a natural number; a ***natnum***.

$\mathbb{N}$ = natural numbers $= \{0, 1, 2, \dots\}$.

$\mathbb{Z}$ = integers $= \{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers $= \{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive ***ratnums*** and $\mathbb{Q}_-$ for the negative ratnums.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

For $\omega \in \mathbb{C}$, let "$\omega > 5$" mean "ω *is* ***real*** and $\omega > 5$". [Use the same convention for $\geqslant, <, \leqslant$, and also if 5 is replaced by any real number.]

An "***interval of integers***" $[b \,..\, c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e} \,..\, 2\pi] = \{3, 4, 5, 6\} = [3 \,..\, 6] = (2 \,..\, 6]$. We allow b and c to be $\pm\infty$; so $(-\infty \,..\, -1]$ is $\mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2i| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand Side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', *but* pt: '*point*', as in "a fixed-pt of a map".

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** fnc, defined for $x>0$, is $\log(x) := \int_1^x \frac{\mathrm{d}v}{v}$. Its inverse-fnc is $\exp()$. For $x>0$, then, $\exp(\log(x)) = x = \mathrm{e}^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(\mathrm{e}^t)$. PolyExp: '*Polynomial-times-exponential*'. E.g, $F(t) := [3 + t^2] \cdot \mathrm{e}^{4t}$ is a polyExp.

Phrases. WLOG: '*Without loss of generality*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. Use iff: '*if and only if*'.

IST: '*It Suffices to*' as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. QED: *quod erat demonstrandum*, meaning "end of proof".

SeLo [2018t] quizzes so far...

P1: Wed. 29 Aug $[\sqrt{3}^{\sqrt{8}}]^{\sqrt{2}} =$ 9 . $\log_{64}(16) =$ $2/3$.

P2: Fri. 31 Aug The four solutions to $[y-2] \cdot y \cdot [y+2] = -1/y$ are $y =$ ....

[*Hint:* Apply the Quadratic Formula to y^2.]

Soln: Cross-multipying and letting $z := y^2$, gives $q(z) = 0$, where $q(z) := z^2 - 4z + 1$. Thus

$$\begin{aligned} \mathrm{Discr}(q) &= [-4]^2 - 4 \cdot 1 \cdot 1 \stackrel{\text{note}}{=\!=} [2 \cdot \sqrt{3}]^2. \quad \textit{Hence,} \\ \mathrm{Roots}(q) &= 2 \pm \sqrt{3}\,. \end{aligned}$$

Consequently, values

$$\pm\sqrt{2 + \sqrt{3}} \quad \textit{and} \quad \pm\sqrt{2 - \sqrt{3}}$$

are the four solns to the original eqn.

P3: Wed. 05 Sep Write the truth-table for $B \Rightarrow [[\neg B] \Rightarrow C]$.

B	C	$\neg B$	$[\neg B] \Rightarrow C$	$B \Rightarrow [[\neg B] \Rightarrow C]$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

P4: Fri. 14 Sep Write the free vars in each of these expressions.

$$\overbrace{\exists n \in \mathbb{N}\colon\ f(n) \subset \underbrace{\bigcup_{\ell=r-4}^{r+7} \underbrace{\{x \in \mathbb{Z} \mid \ell \cdot n \equiv_5 x^2\}}_{E1}}_{E2}}^{E3}$$

$E1$: ℓ, n . $E2$: n, r . $E3$: r, f .

P5: Mon. 17 Sep May lightning ↯ strike this table! (I.e, please fill in.)

n	r_n	q_n	s_n	t_n
0	100	—	**1**	**0**
1	23		**0**	**1**
2				
3				
4				
5				

So GCD(100, 23)= ____ =[____ · 100] + [____ · 23].

LBolt Soln: `(lightning 100 23)`

```
  n:   r_n   q_n   s_n   t_n
/--------------------------\
  0:   100    --     1     0
  1:    23     4     0     1
  2:     8     2     1    -4
  3:     7     1    -2     9
  4:     1     7     3   -13
  5:     0 Infty   -23   100
\__________________________/

  Rename r4,s4,t4  to  GCD,S,T.

  Note    GCD  =  r0 * S  +  r1 * T,  i.e
            1  =  [100]*[3] + [23]*[-13].
```

P6: Mon. 08 Oct LBolt gives $G := \text{GCD}(23, 413) =$ 1 . And $23S + 413T = G$, where $S=$ 18 & $T=$ -1 are integers.

LBolt Soln:

```
  n:   r_n   q_n   t_n   s_n
/--------------------------\
  0:   413    --     1     0
  1:    23    17     0     1
  2:    22     1     1   -17
  3:     1    22    -1    18
  4:     0 Infty    23  -413
\__________________________/

  Rename r3,t3,s3  to  GCD,T,S.

  Note    GCD  =  r0 * T  +  r1 * S,  i.e
            1  =  [413]*[-1] + [23]*[18].
```

Other Bézout pairs are possible.

P7: ⁇ Wed. 10 Oct Solve some of the World's Problems.