Plex
MAA4402 **Quizzes P** Monday 30Aug2021
MAA5404

Number Sets. Expression $k \in \mathbb{N}$ [read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$"] means that k is a natural number; a ***natnum***. Expression $\mathbb{N} \ni k$ [read as "$\mathbb{N}$ ***owns*** k"] is a synonym for $k \in \mathbb{N}$.

$\mathbb{N}$ = natural numbers $= \{0, 1, 2, \ldots\}$.

$\mathbb{Z}$ = integers $= \{\ldots, -2, -1, 0, 1, \ldots\}$. For the set $\{1, 2, 3, \ldots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers $= \{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive rationals and $\mathbb{Q}_-$ for the negative rationals.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

For $\omega \in \mathbb{C}$, let "$\omega > 5$" mean "ω ***is real*** and $\omega > 5$". [Use the same convention for $\geq, <, \leq$, and also if 5 is replaced by any real number.]

Use $\overline{\mathbb{R}} = [-\infty, +\infty] := \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, the ***extended reals***.

An "***interval of integers***" $[b\,..\,c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e}\,..\,2\pi] = \{3, 4, 5, 6\} = [3\,..\,6] = (2\,..\,6]$. We allow b and c to be $\pm\infty$; so $(-\infty\,..\,-1]$ is $\mathbb{Z}_-$. And $[-\infty\,..\,-1]$, is $\{-\infty\} \cup \mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2\mathbf{i}| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). trnfn: '*transformation*'. cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'. RoC: '*Radius of Convergence*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', *but* pt: '*point*' as in "a fixed-pt of a map".

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** fnc, defined for $x>0$, is $\log(x) := \int_1^x \frac{dv}{v}$. Its inverse-fnc is $\exp()$. For $x>0$, then, $\exp(\log(x)) = x = \mathrm{e}^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(\mathrm{e}^t)$.

PolyExp: '*Polynomial-times-exponential*', e.g, $[3 + t^2]\cdot\mathrm{e}^{4t}$. PolyExp-sum: '*Sum of polyexps*'. E.g, $f(t) := 3t\mathrm{e}^{2t} + [t^2]\cdot\mathrm{e}^t$ is a polyexp-sum.

Phrases. WLOG: '*Without loss of generality*'. IFF: '*if and only if*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. And ※="Contradiction".

IST: '*It Suffices To*', as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. inter alia: '*among other things*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Sequence notation. A *sequence* $\vec{\mathbf{x}}$ abbreviates $(x_0, x_1, x_2, x_3, \ldots)$. For a set Ω, expression "$\vec{\mathbf{x}} \subset \Omega$" means $[\forall n\colon x_n \in \Omega]$. Use $\mathrm{Tail}_N(\vec{\mathbf{x}})$ for the subsequence

$$(x_N, x_{N+1}, x_{N+2}, \ldots)$$

of $\vec{\mathbf{x}}$. Given a fnc $f\colon\Omega\to\Lambda$ and an Ω-sequence $\vec{\mathbf{x}}$, let $f(\vec{\mathbf{x}})$ be the Λ-sequence $(f(x_1), f(x_2), f(x_2), \ldots)$.

Suppose Ω has an addition and multiplication. For Ω-seqs $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$, then, let $\vec{\mathbf{x}} + \vec{\mathbf{y}}$ be the sequence whose n^{th} member is $x_n + y_n$. I.e

$$\vec{\mathbf{x}} + \vec{\mathbf{y}} = [n \mapsto [x_n + y_n]].$$

Similarly, $\vec{\mathbf{x}} \cdot \vec{\mathbf{y}}$ denotes seq $[n \mapsto [x_n \cdot y_n]]$.

Plex notation. Let *SCC* mean "positively oriented simple-closed-contour". For a SCC C, have $\mathring{C}$ be the (open) region C encloses, and let $\widehat{C}$ mean C together with $\mathring{C}$. So $\widehat{C}$ is $C \sqcup \mathring{C}$; it is automatically simply-connected and is a closed bounded set.

Plex [2021t] quizzes so far...

P1: Wed. 01 Sep Blanks$\in\mathbb{R}$. So $\frac{1}{2+3\mathbf{i}} =$ ______ $+\mathbf{i}\cdot$[______].

Thus $\dfrac{5-\mathbf{i}}{2+3\mathbf{i}} =$ ______ $+\mathbf{i}\cdot$[______].

Soln: Note $\omega := 2+3\mathbf{i}$ is not zero. Thus $\frac{1}{\omega} = \frac{\overline{\omega}}{\omega\overline{\omega}} = \frac{\overline{\omega}}{|\omega|^2}$. So

$$\frac{1}{\omega} = \frac{\mathrm{Re}(\omega)}{|\omega|^2} + \mathbf{i}\cdot\frac{-\mathrm{Im}(\omega)}{|\omega|^2} = \frac{2}{13} + \mathbf{i}\cdot\frac{-3}{13}.$$

With $\alpha := 5-\mathbf{i}$, note $|\omega|^2\cdot\frac{\alpha}{\omega} = \overline{\omega}\alpha = 7-17\mathbf{i}$. Whence

$$\frac{\alpha}{\omega} = \frac{7}{13} + \mathbf{i}\frac{-17}{13}.$$

P2: Mon. 11 Oct With $\alpha := [3\mathbf{i}]^3$ and $x+\mathbf{i}y := \mathrm{Log}(\alpha)$, then

$x=$ $3\log(3)$ *and* $y=$ $\frac{-\pi}{2}$.

Principal Val. Note $|\alpha| = |{-\mathbf{i}}\cdot 3^3| = 3^3$. And $\mathrm{Arg}(\alpha) = \frac{-\pi}{2}$. Thus $\mathrm{Log}(\alpha) = \log(3^3) - \frac{\pi}{2}\mathbf{i} = 3\log(3) - \frac{\pi}{2}\mathbf{i}$. ♦

P3: Wed. 13 Oct [Below, "Log" is the P.V. of $\mathbb{C}$-logarithm, and $z,w\in\mathbb{C}$.]

If $\mathrm{Re}(z)<0$ then $\mathrm{Log}(z^2) = 2\,\mathrm{Log}(z)$. AT **[AF]** Nei

If $\mathrm{Re}(z)>1$ then $\mathrm{Log}(z^3) = 3\,\mathrm{Log}(z)$. AT AF **[Nei]**

If $\mathrm{Re}(z)=0$ and $\mathrm{Im}(z)>2$ then $\mathrm{Log}(\exp(z)) = z$. AT AF **[Nei]**

If $\mathrm{Re}(z)>0$ and $\mathrm{Re}(w)>0$ then $\mathrm{Log}(z\cdot w) = \mathrm{Log}(z)+\mathrm{Log}(w)$. **[AT]** AF Nei

So $\mathrm{Log}([1+\mathbf{i}]^{13}) = 13\ln(\sqrt{2}) + y\mathbf{i}$, where $y = -\frac{3}{4}\cdot\pi$.

Rem. The hypotheses below force $z,w\neq 0$, so we may write $z = re^{\mathbf{i}\theta}$ and $w = Re^{\mathbf{i}\varphi}$, with $r,R>0$ and $\theta,\varphi\in(-\pi,\pi]$. □

a Soln. Note $\mathrm{Im}(2\,\mathrm{Log}(z)) = 2\theta$. Hypothesis $\mathrm{Re}(z)<0$ says $\theta\in(\frac{\pi}{2},\pi]$ *or* $\theta\in(-\pi,-\frac{\pi}{2})$. In the former case, $2\theta>\pi$, hence $\mathrm{Im}(\mathrm{Log}(z^2)) = 2\theta-2\pi$. In the latter case, $2\theta<-\pi$ whence $\mathrm{Im}(\mathrm{Log}(z^2)) = 2\theta+2\pi$. Consequently, *AF*. ♦

b Soln. As above, $\mathrm{Im}(3\,\mathrm{Log}(z)) = 3\theta$. Constraint $\mathrm{Re}(z)>1$ says $\theta\in(-\frac{\pi}{2},\frac{\pi}{2})$. [The same results from $\mathrm{Re}(z)>0$.] Hence $\mathrm{Log}(z^3)$ equals 3θ *exactly when* $\theta\in(\frac{-\pi}{3},\frac{\pi}{3}]$. Thus *Nei*. ♦

c Soln. In general, $\mathrm{Log}(\exp(z)) = z$ IFF $-\pi<\mathrm{Im}(z)\leq\pi$. The given constraint is $\mathrm{Im}(z)\in(2,\infty)$, which has values both in and out of $(-\pi,\pi]$, since $2<\pi$. Hence *Nei*. ♦

d Soln. Constraint $\mathrm{Re}(z),\mathrm{Re}(w)>0$ is equivalent to $\theta,\varphi\in(-\frac{\pi}{2},\frac{\pi}{2})$. Thus $\theta+\varphi\in(-\pi,\pi)$, so $\mathrm{Arg}(z\cdot w)$ indeed equals $\mathrm{Arg}(z)+\mathrm{Arg}(w)$. Upshot: *AT*. ♦

e Soln. Since $\mathrm{Arg}(1+\mathbf{i}) = \frac{1}{4}\cdot\pi$, we have that $\mathrm{Arg}([1+\mathbf{i}]^{13}) = [\frac{13}{4}-2n]\cdot\pi$, for *the* integer n that puts $\frac{13}{4}-2n$ in $(-1,1]$. Happily, $\frac{13}{4} = 4-\frac{3}{4}$, so $y = -\frac{3}{4}\pi$. ♦

P4: $^{\text{Fri.}}_{\text{15 Oct}}$ With L the line-segment contour from $\mathbf{p} := 4$ to $\mathbf{q} := 7+\mathbf{i}$, compute $J := \int_{\mathsf{L}} \overline{z}\, dz$, showing the parametrization you used and resulting definite integral.

Soln. With $\mathsf{D} := \mathbf{q} - \mathbf{p} = 3+\mathbf{i}$, the constant-speed param is $\lambda(t) := \mathbf{p} + \mathsf{D}t = 4 + [3+\mathbf{i}]t$, for $0 \le t \le 1$. Thus J equals

$$\int_0^1 \overline{\lambda(t)} \cdot \lambda'(t)\, dt = \mathsf{D} \int_0^1 \overline{\mathbf{p} + \mathsf{D}t}\, dt = \mathsf{D} \int_0^1 \left[4 + \overline{\mathsf{D}}t\right] dt = \mathsf{D} \cdot \left[4 + \frac{\overline{\mathsf{D}}}{2}\right] = \mathsf{D} \cdot [8 + \overline{\mathsf{D}}] \cdot \frac{1}{2}.$$

Multiplying, $2J = [3+\mathbf{i}] \cdot [11 - \mathbf{i}] = 34 + 8\mathbf{i}$, revealing that $J = 17 + 4\mathbf{i}$. ♦

P5: $^{\text{Fri.}}_{\text{22 Oct}}$ Let $\mathsf{C} := \mathrm{Sph}_3(\mathbf{i})$, a radius 3 circle centered at $\mathbf{i}$. Integral

$$T := \oint_{\mathsf{C}} \frac{\cos(z^2)}{1 - 3z}\, dz = \text{.............................}.$$

[*Hint:* Use the Cauchy Integral Formula.]

Soln: Note $T = \frac{1}{-3} J$, where

$$J := \oint_{\mathsf{C}} \frac{\cos(z^2)}{z - \frac{1}{3}}\, dz \overset{\text{by CIF}}{=\!=} 2\boldsymbol{\pi}\mathbf{i} \cdot \cos\left([1/3]^2\right),$$

since $\frac{1}{3} \in \mathring{\mathsf{C}}$ and $\cos(z^2)$ is holomorphic. Thus

$$T = \frac{-2\boldsymbol{\pi}\mathbf{i}}{3} \cdot \cos(1/9).$$

P6: Mon. 25 Oct Let R be the pos-oriented rectangle with corners $2 \pm \mathbf{i}$ and $-9 \pm \mathbf{i}$.

Then $T := \oint_{\mathsf{R}} \frac{\exp(2z)}{z^2 - 9}\,\mathrm{d}z =$ ______________________.

Soln: The poles of the integrand are $\mathbf{p} := -3$ and $\mathbf{q} := 3$. Only $\mathbf{p}$ is enclosed by R, so $h(z) := \frac{\exp(2z)}{z-3}$ is holomorphic on $\widehat{\mathsf{R}}$. Thus CIF applies, asserting

$$\frac{T}{2\pi\mathbf{i}} = \frac{1}{2\pi\mathbf{i}} \oint_{\mathsf{R}} \frac{h(z)}{z+3}\,\mathrm{d}z \overset{\mathsf{CIF}}{=\!=} h(-3) = \frac{\exp(-6)}{-6}.$$

Consequently, $$T = \frac{-\pi\mathbf{i}}{3\,\mathrm{e}^6}.$$

Let $\mathsf{C} := \mathrm{Sph}_3(\mathbf{i})$, a radius 3 circle centered at $\mathbf{i}$. Integral $\oint_{\mathsf{C}} \frac{\cos(z^2)}{3-z}\,\mathrm{d}z = 0$, since integrand is holomorphic.

... except at 3. But $3 \notin \widehat{\mathsf{C}} \overset{\text{recall}}{=\!=} \mathrm{CldBal}_3(\mathbf{i})$, so Cauchy-Goursat says zero.

P: Wed. 27 Oct *Class-W, in-class exam. Bring lots of paper, and perhaps colored pencils for drawing pictures.*

P7: Fri. 29 Oct Let $f(x+\mathbf{i}y) := 4x^3 + \mathbf{i}y^3$. Let $\mathbf{D},\mathbf{H},\mathbf{C} \subset \mathbb{C}$ be the sets where f is, respectively: **D***ifferentiable*, **H***olomorphic*, *has* **C***ontinuous 1st-partials.* Then

D = ... ;

H = ... ;

C = ...

[You may use: *Set-builder notation. List pts between braces. Union & intersection symbols. Sets* $\mathbb{C}$, $\mathbb{R}$, $\varnothing$.]

Soln **C**. Decompsing $f = \mathsf{u} + \mathbf{i}\mathsf{v}$ yields $\mathsf{u} = 4x^3$ and $\mathsf{v} = y^3$, which hands us 1st-partials

†: $\mathsf{u}_y = 0 = -0 = -\mathsf{v}_x$, *and*

‡: $\mathsf{u}_x = 3 \cdot 4x^2$; $3 \cdot y^2 = \mathsf{v}_y$.

All of these are cts fncs of (x, y), hence $\boxed{\mathbf{C} = \mathbb{C}}$. ♦

Soln **D**. Recall our theorem:

> *Suppose f has 1st-partials defined in a nbhd of a point* $\mathbf{p}$*, and* *continuous* *at* $\mathbf{p}$*. Then f is* (complex-)*differentiable at* $\mathbf{p}$ *IFF f satisfies the Cauchy-Riemann eqns at* $\mathbf{p}$.

[See: FIRST COURSE 2.13, P.26. *B&C*, P.66. PLEXNOTES Suff. cond. for Differentiability, ≈P.56.] Well, C-R-(†) always holds. The other C-R eqn is equality in (‡), that is, $[2x]^2 = y^2$. Thus $\boxed{\mathbf{D} = \{(x, \pm 2x) \mid x \in \mathbb{R}\}}$, a union of two lines. ♦

Soln **H**. Recall: *Fnc g is* ***"holomorphic** at* $\mathbf{p}$*" if g is defined and differentiable in a* *nbhd* *of* $\mathbf{p}$. But our above set **D** has *empty* (topological-)interior. Hence $\boxed{\mathbf{H} = \varnothing}$.♦

P8: Fri. 05 Nov Compute $J := \oint_{\mathsf{C}} \frac{\cos(2z)}{[z-4][z-100]} \,\mathrm{d}z$, where $\mathsf{C} := \mathrm{Sph}_9(\mathbf{i})$, the radius-9 circle centered at $\mathbf{i}$.

CIF to the rescue. Poles of the integrand are $4 \overset{\text{note}}{\in} \mathring{\mathsf{C}}$ and $100 \notin \widehat{\mathsf{C}}$. So CIF applies to $g(z) := \frac{\cos(2z)}{z-100}$, asserting

$$\frac{J}{2\pi\mathbf{i}} = \frac{1}{2\pi\mathbf{i}} \oint_{\mathsf{C}} \frac{g(z)}{z-4} \,\mathrm{d}z = g(4)\,.$$

Thus $J = 2\pi\mathbf{i} \cdot \frac{\cos(2\cdot 4)}{4 - 100} = \frac{-\pi \cdot \cos(8)}{48} \cdot \mathbf{i}$. ♦

PBonus: Mon. 08 Nov *Our W-Bonus was due at the Beginning of Class, hopeully nicely typeset and well-written.*

P9: Wed. 10 Nov Prof. K thinks the flu vaccine is *Good Idea*, even if PUBLIX offers no gift-card. [Circle]: (Yes) (True)

The visual representation of $\mathbb{C}$ is sometimes called "the ? plane", where ? is [Circle]: **Unreal** **Higher** **Snakes-on-a** **(Argand)** **Krypton** **Rayon** **Xenon** **Euler** **Goursat** **Liouville** **No-need-to-x** **y-com** **Air** **Sea** **De** **Rain-in-Spain-stays-mainly-on-the** .

PA: Fri. 12 Nov Let $f(z) := z^4 \exp(2/z)$.
Then $\text{Res}(f, 0) =$ ....

[Hint: *Write the PS for* e^w, *then plug in* $2/z$ *for* w. *Multiply the resulting Laurent Series by* z^4. *You may use the factorial symbol in expressing your answer.*]

Power to the Series! By definition of $\exp()$,

$$\exp(2/z) = \sum_{n=0}^{\infty} \frac{[2/z]^n}{n!} = \sum_{n=0}^{\infty} \frac{2^n}{n!} \cdot z^{-n}. \quad \textit{So,}$$

$$f(z) = z^4 \cdot \text{PS} = \sum_{n=0}^{\infty} \frac{2^n}{n!} \cdot z^{4-n}.$$

The coefficient of z^{-1} is at $n=5$, whence

$$\text{Res}(f, 0) = \frac{2^5}{5!} = \frac{4}{15}.$$

♦

A *minimum* requirement for an LOR (letter-of-recommendation) from Prof. K is <u>two</u> courses. Circle:

Yes **True** **Darn tootin'*!***

PX: Wed. 17 Nov *Class-X*, our third in-class exam.

PB: Fri. 19 Nov Let $f(z) := z^4 + 13z^2 + 36$. Reciprocal $H(z) := 1/f(z)$ has, in the upper half-plane, two poles $\mathbf{p}$ and $\mathbf{q}$, where $\mathbf{p}$ lies closer to the origin than $\mathbf{q}$.

So $\operatorname{Res}(H, \mathbf{p}) =$ $1/[20\,\mathbf{i}]$ *and* $\operatorname{Res}(H, \mathbf{q}) =$ $-1/[30\,\mathbf{i}]$.

Our D-contour technique applies to H.

Thus $J := \int_{-\infty}^{+\infty} \frac{1}{z^4 + 13z^2 + 36}\,dx =$ $\pi/30$.

Two poles. Factoring, $f(z) = [z^2+4][z^2+9]$. Thus, the upper half-plane poles are $\mathbf{p} = 2\mathbf{i}$ and $\mathbf{q} = 3\mathbf{i}$. As these are simple poles, CIF hands us

$$\operatorname{Res}(H, \mathbf{p}{=}\mathbf{i}) = \frac{1}{[z^2+9][z+2\mathbf{i}]}\bigg|_{z=2\mathbf{i}} = \frac{1}{5 \cdot 4\mathbf{i}} \quad \textit{and}$$

$$\operatorname{Res}(H, \mathbf{q}{=}2\mathbf{i}) = \frac{1}{[z^2+4][z+3\mathbf{i}]}\bigg|_{z=3\mathbf{i}} = \frac{1}{-5 \cdot 6\mathbf{i}}.$$

The arc-integral $\int_{\mathsf{A}_r} \frac{1}{f}$ goes to zero as $r \nearrow \infty$. Hence

$$J = 2\pi\mathbf{i} \cdot \textit{Sum-of-residues} = 2\pi\mathbf{i} \cdot \left[\frac{1}{20\,\mathbf{i}} - \frac{1}{30\,\mathbf{i}}\right] = \frac{\pi}{30}. \quad ♦$$

Prof. King believes that writing in complete, coherent sentences is crucial in communicating Mathematics, improves posture, and whitens teeth. Circle one:

True! **Yes!** **wH'at S a?sEnTENcE**

PC: Mon. 22 Nov STMT: Most folks like Tnanksgiving Break. (Circle)

T *F* *What? We don't have class?*

A function holomorphic on open $\operatorname{Bal}_2(0)$

with poles at $\pm 2\mathbf{i}$ *and* at ± 2 is $h(z) :=$ ______.

Soln. Simplest is $h(z) := \frac{1}{z^4 - 16}$. ♦

PThanksgiving: Wed-Sat. 24-27 Nov *No classes, so ... Lots of time to post solutions to our Archive!*

PD: Mon. 29 Nov The IOP (Individual Optional Project), if you choose to do it, is due by 2PM on Thursday, 09Dec2021, slid *completely* under my office door, Little Hall 402 (northeast corner of top floor) Circle: Yes Cool! Thanks

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On annulus $\text{Ann}_2^{\infty}(0)$, function $f(z) := 1/[z - 2\mathbf{i}]$ has Laurent series $\sum_{n=-\infty}^{\infty} b_n z^n$, where $b_{-4} =$ ______, $b_{-3} =$ ______, $b_0 =$ ______, and $b_2 =$ ______.

Laurentify. Rewrite as $f(z) = \dfrac{1/z}{1 - \frac{2\mathbf{i}}{z}}$. Expanding the geometric series,

$$\ddagger: \quad f(z) = \frac{1}{z}\sum_{n=0}^{\infty}\left[\frac{2\mathbf{i}}{z}\right]^n = \sum_{k=1}^{\infty}[2\mathbf{i}]^{k-1}\cdot z^{-k}.$$

So $b_0 = 0 = b_2$. And $b_{-4} = 2^3 \cdot \mathbf{i}^3 = -8\mathbf{i}$. Lastly $b_{-3} = 2^2 \cdot \mathbf{i}^2 = -4$. ♦

PE: Wed. 01 Dec Residue $\operatorname{Res}_{z=0}\left(z^6 \cdot \sin\left(\frac{1}{3z}\right)\right) =$ ______.

Sine language. Since $\sin(w) = \sum_{k=0}^{\infty} \frac{[-1]^k}{[2k+1]!} \cdot w^{2k+1}$,

$$\sin\left(\frac{1}{3z}\right) = \sum_{k=0}^{\infty} \frac{[-1]^k}{[2k+1]!} \cdot \frac{1}{[3z]^{2k+1}}.$$

Index $k=3$ gives the z^{-7} term, so the coefficient of z^{-7} is

$$\frac{-1}{7! \cdot 3^7}.$$

Thus this, in product $z^6 \cdot \sin\left(\frac{1}{3z}\right)$, is the coeff of z^{-1}, hence is the residue we sought. ♦

PF: Wed. 08 Dec Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, $500 total] is a *really good idea*. A ten-page essay is fine. Date for the emailed-PDF is Sunday, March 27, 2022.

Circle: **Yes** **True** ***Résumé material!***