

Number Sets. Expression $k \in \mathbb{N}$ [read as “ k is an element of \mathbb{N} ” or “ k in \mathbb{N} ”] means that k is a natural number; a **natnum**. Expression $\mathbb{N} \ni k$ [read as “ \mathbb{N} owns k ”] is a synonym for $k \in \mathbb{N}$.

\mathbb{N} = natural numbers = $\{0, 1, 2, \dots\}$.

\mathbb{Z} = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the **posints**, use \mathbb{Z}_+ . Use \mathbb{Z}_- for the negative integers, the **negints**.

\mathbb{Q} = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use \mathbb{Q}_+ for the positive rationals and \mathbb{Q}_- for the negative rationals.

\mathbb{R} = reals. The **posreals** \mathbb{R}_+ and the **negreals** \mathbb{R}_- .

\mathbb{C} = complex numbers, also called the **complexes**.

For $\omega \in \mathbb{C}$, let “ $\omega > 5$ ” mean “ ω is real and $\omega > 5$ ”.

[Use the same convention for $\geq, <, \leq$, and also if 5 is replaced by any real number.]

Use $\overline{\mathbb{R}} = [-\infty, +\infty] := \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, the **extended reals**.

An “**interval of integers**” $[b..c]$ means the intersection $[b, c] \cap \mathbb{Z}$; ditto for open and closed intervals. So $[e..2\pi] = \{3, 4, 5, 6\} = [3..6] = (2..6]$. We allow b and c to be $\pm\infty$; so $(-\infty..-1]$ is \mathbb{Z}_- . And $[-\infty..-1]$, is $\{-\infty\} \cup \mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2i| = \sqrt{29}$.

Mathematical objects. Seq: ‘sequence’. poly(s): ‘polynomial(s)’. irred: ‘irreducible’. Coeff: ‘coefficient’ and var(s): ‘variable(s)’ and parm(s): ‘parameter(s)’. Expr.: ‘expression’. Fnc: ‘function’ (so ratfnc: means rational function, a ratio of polynomials). trnfn: ‘transformation’. cty: ‘continuity’. cts: ‘continuous’. diff’able: ‘differentiable’. CoV: ‘Change-of-Variable’. Col: ‘Constant of Integration’. Lol: ‘Limit(s) of Integration’. RoC: ‘Radius of Convergence’.

Soln: ‘Solution’. Thm: ‘Theorem’. Prop’n: ‘Proposition’. CEX: ‘Counterexample’. eqn: ‘equation’. RhS: ‘RightHand side’ of an eqn or inequality. LhS: ‘lefthand side’. Sqrt or Sqroot: ‘square-root’, e.g., “the sqroot of 16 is 4”. Ptn: ‘partition’, but pt: ‘point’ as in “a fixed-pt of a map”.

FTC: ‘Fund. Thm of Calculus’. IVT: ‘intermediate-Value Thm’. MVT: ‘Mean-Value Thm’.

The **logarithm** fnc, defined for $x > 0$, is $\log(x) := \int_1^x \frac{dv}{v}$. Its inverse-fnc is $\exp()$. For $x > 0$, then, $\exp(\log(x)) = x = e^{\log(x)}$. For real t , naturally, $\log(\exp(t)) = t = \log(e^t)$.

PolyExp: ‘Polynomial-times-exponential’, e.g., $[3 + t^2] \cdot e^{4t}$. PolyExp-sum: ‘Sum of polyexps’. E.g., $f(t) := 3te^{2t} + [t^2] \cdot e^t$ is a polyexp-sum.

Phrases. WLOG: ‘Without loss of generality’. IFF: ‘if and only if’. TFAE: ‘The following are equivalent’.

ITOf: ‘In Terms Of’. OTForm: ‘of the form’. FTSOC: ‘For the sake of contradiction’. And \otimes = “Contradiction”.

IST: ‘It Suffices To’, as in ISTShow, ISTExhibit.

Use w.r.t: ‘with respect to’ and s.t: ‘such that’.

Latin: e.g. *exempli gratia*, ‘for example’. i.e. *id est*, ‘that is’. N.B: *Nota bene*, ‘Note well’. inter alia: ‘among other things’. QED: *quod erat demonstrandum*, meaning “end of proof”.

Sequence notation. A sequence \vec{x} abbreviates $(x_0, x_1, x_2, x_3, \dots)$. For a set Ω , expression “ $\vec{x} \subset \Omega$ ” means $[\forall n: x_n \in \Omega]$. Use $\text{Tail}_N(\vec{x})$ for the subsequence

$$(x_N, x_{N+1}, x_{N+2}, \dots)$$

of \vec{x} . Given a fnc $f: \Omega \rightarrow \Lambda$ and an Ω -sequence \vec{x} , let $f(\vec{x})$ be the Λ -sequence $(f(x_1), f(x_2), f(x_3), \dots)$.

Suppose Ω has an addition and multiplication. For Ω -seqs \vec{x} and \vec{y} , then, let $\vec{x} + \vec{y}$ be the sequence whose n^{th} member is $x_n + y_n$. I.e

$$\vec{x} + \vec{y} = [n \mapsto [x_n + y_n]].$$

Similarly, $\vec{x} \cdot \vec{y}$ denotes seq $[n \mapsto [x_n \cdot y_n]]$.

LinA quizzes during Add/Drop

These do not count for a grade.

DNC1: ^{Mon.}_{10 Jan} The **slope** of line $3[y - 5] = 2[x - 2]$ is _____.

Point $(-4, y)$ lies on this line, where $y =$ _____.

Quadratic $15x^2 + 23x + 6 = [Ax - \alpha] \cdot [Bx - \beta]$, for numbers $A =$ _____, $\alpha =$ _____; $B =$ _____, $\beta =$ _____.

Q3: ^{Fri.}_{14 Jan} Matrix-product $\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 7 & w \\ 2 & 1 \end{bmatrix} =$ _____.

Q4: ^{Tue.}_{18 Jan} Blanks $\in \mathbb{R}$. So $\frac{1}{2+3i} =$ _____ $+ i \cdot$ [_____].

Thus $\text{Im}\left(\frac{5-i}{2+3i}\right) =$ _____.

By the way, $|5 - 3i| =$ _____.

LinA [2022g] quizzes

These count!

(Recall, the lowest MQ score is dropped. In consequence, there is no make-up for the first missed MQ.)

Q1: ^{Tue.}_{11 Jan} Jake Park's office-hour desk is in

Narnia **Oz** **Lit403(NE)**

Line $y = Mx + B$ is orthogonal to $y = \frac{1}{3}x + 2$ and owns $(4, 10)$. So $M =$ _____ and $B =$ _____.

Q2: ^{Wed.}_{12 Jan} The four solutions to $[y - 2] \cdot y \cdot [y + 2] = -1/y$ are $y =$ _____.

[Hint: Apply the Quadratic Formula to y^2 .]

Q5: ^{Tue.}_{01 Feb} Collection $\mathcal{B} := \{x^3 - 2x^2 + 1, 4x^2 - x + 3, 3x - 2\}$

generates VS $P_3(\mathbb{R})$. T F

In VS \mathbf{V} , collection $\mathcal{C} := \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_8\}$ is LD. Then each \mathbf{v}_j lies in $\text{Spn}(\mathcal{C} \setminus \{\mathbf{v}_j\})$. T F

Q6: ^{Fri.}_{11 Feb} VS $\mathbf{V} := \text{MAT}_{4 \times 4}(\mathbb{R})$ is a 16-dim'al \mathbb{R} -VS.
 Define lin.trn $D: \mathbf{V} \rightarrow \mathbf{V}$ by $D(M) := M + M^t$. Then
 Nullity(D) = And Rank(D) =

The trn $U: \mathbf{V} \rightarrow \mathbf{V}$ by $U(M) := M^2$ is:
 Linear Affine (but not linear) Not-affine

Q7: ^{Fri.}_{18 Feb} Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ rotate the plane CCW by 60° , then vertically stretch by a factor of 8. W.r.t the std basis,

$$[[T]]_{\mathcal{E}}^{\mathcal{E}} = \left[\begin{array}{c|c} & \\ \hline & \end{array} \right].$$

Q8: ^{Tue.}_{22 Feb} Suppose $T: \mathbf{V} \rightarrow \mathbf{W}$ is a VS-isomorphism between finite dim'al VSes. Subspace $\mathbf{V}_0 \subset \mathbf{V}$ has dimension m . Then $\text{Dim}(T(\mathbf{V}_0)) = \dots\dots\dots$.

Q9: ^{Fri.}_{25 Feb} Over field \mathbb{Z}_7 , matrix $M := \begin{bmatrix} 2 & 0 & 3 \\ 0 & 0 & -1 \\ 0 & -2 & 2 \end{bmatrix}$ has

inverse

$$M^{-1} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}.$$

[Write your answer using *symmetric* residues.]

QA: ^{Tue.}_{01 Mar} $\text{Det}(A) = \underline{\hspace{2cm}}$ and $\text{Det}(B) = \underline{\hspace{2cm}}$,
for \mathbb{Q} -matrices

$$A := \begin{bmatrix} 19 & 22 & 7 \\ 0 & 8 & 0 \\ 0 & 9 & 0 \end{bmatrix} \quad \text{and} \quad B := \begin{bmatrix} 0 & 11 & 0 \\ 12 & 2 & 7 \\ 0 & 3 & 0 \end{bmatrix}.$$

QB: ^{Fri.}_{04 Mar} Pr.K thinks *Spring Break* is an opportunity to start a Robert Long essay. Circle **TrueDarn tootin'!**

Over field \mathbb{Z}_{11} , matrix $C := \begin{bmatrix} 0 & 0 & -2 \\ 0 & 3 & 0 \\ 4 & -2 & -3 \end{bmatrix}$ has inverse

$$C^{-1} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}.$$

[Write your answer using *symmetric* residues.]

QC: ^{Tue.}_{29 Mar} For matrix $A := \begin{bmatrix} -5 & 2 \\ 0 & -4 \end{bmatrix}$, the eVal corresponding to $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is

QD: ^{Tue.}_{12 Apr} Let $A := \begin{bmatrix} 1+i & 3+4i & 7 \\ 1+2i & 5i & 2+i \end{bmatrix}$. Then A^* is a \times matrix. In order, the entries in the top row of A^* are

QE: ^{Tue.}_{19 Apr} Let $A := \begin{bmatrix} i & i \\ i & -i \end{bmatrix}$. Compute A^* , as well as products AA^* and A^*A , to conclude that A is *normal*, that is, that A commutes with its adjoint.