LinA MAS4105 **Quizzes Q** Wednesday 09Sep2015

Number Sets. An expression such as $k \in \mathbb{N}$ (read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$") means that k is a natural number; a ***natnum***.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \dots\}$.

$\mathbb{Z}$ = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive ***ratnums*** and $\mathbb{Q}_-$ for the negative ratnums.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

An **"*interval of integers*"** $[b \,..\, c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e} \,..\, 2\pi] = \{3, 4, 5, 6\} = [3 \,..\, 6] = (2 \,..\, 6]$. We allow b and c to be $\pm\infty$; so $(-\infty \,..\, -1]$ is $\mathbb{Z}_-$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'.

Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand Side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', *but* pt: '*point*', as in "a fixed-pt of a map".

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'. CoV: '*Change-of-Variable*'.

Phrases. WLOG: '*Without loss of generality*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. Use iff: '*if and only if*'.

IST: '*It Suffices to*' as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. QED: *quod erat demonstrandum*, meaning "end of proof".

LinA quizzes so far...

Q0: Mon. 31,Aug The **slope** of line $3[y-5] = 2[x-2]$ is ______.
Point $(-4, y)$ lies on this line, where y= ______.

Let $y = f(x) := [5 + \sqrt[3]{x}]/2$. Its inverse-function is $f^{-1}(y)$= ______.

Q1: Wed. 09 Sep *Henceforth*, let *AT* mean "Always True", *AF* mean "Always False" and *Nei* mean "Neither always true nor always false". Below, $\mathbf{u}, \mathbf{v}, \mathbf{w}$ repr. *distinct, non-zero* vectors in $\mathbb{R}^4$, a $\mathbb{R}$-VS. Please circle the correct response:

y1 If $\mathbf{w} \in \mathrm{Spn}\{\mathbf{u}, \mathbf{v}\}$ then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. *AT AF Nei*

y2 If $\mathbf{w} \notin \mathrm{Spn}\{\mathbf{u}, \mathbf{v}\}$ then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. *AT AF Nei*

y3 Collection $\{\mathbf{0}, \mathbf{w}\}$ is linearly-indep. *AT AF Nei*

y4 $\mathrm{Spn}\{\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{u} + 2\mathbf{v} + 3\mathbf{w}\}$ is all of $\mathbb{R}^4$. *AT AF Nei*

y5 If none of $\mathbf{u}, \mathbf{v}, \mathbf{w}$ is a multiple of the other vectors, then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. *AT AF Nei*

Q2: Fri. 18 Sep A system of 3 linear equations in unknowns $x_1, \dots, x_5$ reduces to the augmented matrix

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 1 & 12 \\ 0 & 0 & 1 & 0 & -8 & 34 \\ 0 & 0 & 0 & 1 & 5 & -56 \end{array}\right],$$

which is in RREF. Please circle each *pivot entry.*

OYOP, describe the *general solution* in this form,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \alpha \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \beta \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \gamma \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \dots$$

where each $\alpha, \beta, \gamma, \delta, \dots$ is a free variable (either x_1 or... or x_5), and each column vector has specific numbers in it. Dim(*SolnFlat*) = ______.

Q3: Wed. 30Sep Inverse of $\begin{bmatrix} 3 & 1 & 0 \\ 5 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is

.

Q4: Mon. 05Oct Let R_θ be the std. rotation [by θ] matrix. With

$$\mathsf{C} := \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix} \quad \text{and} \quad \mathsf{B} := \begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix},$$

the product $[\mathsf{CB}]^{22} = \alpha \cdot \mathsf{R}_\theta$, with $\alpha=$ ________ $\in\mathbb{R}_+$ and $\theta=$ ________ $\in(-180°, 180°]$. [*Hint:* Don't multiply matrices. Geometrically, C and B represent what linear-trns?]

Q5: Tue. 06Oct Let $\mathsf{T}:\mathbb{R}^2 \circlearrowleft$ rotate the plane CCW by 60°, then vertically stretch by a factor of 5. W.r.t the std basis,

$$[\![\mathsf{T}]\!]_{\mathcal{E}}^{\mathcal{E}} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}.$$

Q6: Tue. 20Oct Let $\mathsf{T}:\mathbb{R}^2\to\mathbb{R}^2$ by $\mathsf{T}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) := \begin{bmatrix} 3x-y \\ 2x+6y \end{bmatrix}$. W.r.t ordered-basis $\mathcal{B} := \left(\begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix}\right)$, let $\mathsf{M} := [\![\mathsf{T}]\!]_{\mathcal{B}}^{\mathcal{B}}$. Then $\mathsf{M}=\mathsf{RTR}^{-1}$,

where $\mathsf{R} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$, $\mathsf{M} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$.

Q7: Tue. 20Oct In $\mathbb{R}^3$, the closest point to $\mathbf{v} := (1,2,3)$ on the line through $\mathbf{0}$ and $\mathbf{q} := (-2,7,0)$, is $\alpha\mathbf{q}$, where $\alpha=$ ______

In $\mathbb{R}^2$, with $\mathbf{s} := (1,8)$ and $\mathbf{w} := (4,-2)$, compute $\text{Orth}_{\mathbf{w}}(\mathbf{s})=$ ________.

Q8: Mon. 30Nov Let $\mathsf{B} := \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 3 & 6 & 0 & -3 & 0 \end{bmatrix}$. Then $\mathsf{R} := \mathit{RREF}(\mathsf{B})$ is [show no work, here]

$$\mathsf{R} = \begin{bmatrix} \quad & \quad & \quad & \quad & \quad \end{bmatrix}.$$

I For subspace $\mathbf{V} := \text{Nul}(\mathsf{L_B})$, use back-substitution, and *scaling*, to produce an *integer basis*

$\mathbf{v}_1:=($ __ , __ , __ , __ , __ $)$ $\mathbf{v}_2:=($ __ , __ , __ , __ , __ $)$,

$\mathbf{v}_3:=($ __ , __ , __ , __ , __ $)$, $\mathbf{v}_4:=($ __ , __ , __ , __ , __ $)$.

[*Note:* **Only use as many as the dimension of V.**]

II Apply Gram-Schmidt to compute an **orthogonal** *integer-basis* for $\mathbf{V}$:

$\mathbf{b}_1:=($ __ , __ , __ , __ , __ $)$, $\mathbf{b}_2:=($ __ , __ , __ , __ , __ $)$,

$\mathbf{b}_3:=($ __ , __ , __ , __ , __ $)$, $\mathbf{b}_4:=($ __ , __ , __ , __ , __ $)$.

[Entries are integers]

Arrange that the Gcd of the entries in each vector is 1, and that the first non-zero value is positive.

Q9: Tue. 08Dec Line $y = \big[[$ ______ $]x\big] +$ ______ is the least-squares best-fit to data pts $\{(-2,0), (-1,0), (1,0), (2,1)\}$.

That's All, Folks!