Lina
MAS4105

Quizzes Q

29Sep2010

Q1: Tues. 6 Sep Matrix-product $\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 7 & w \\ 2 & 1 \end{bmatrix} =$ ....

Matrix-add distributes over matrix-mult: .... *T* *F*

Q2: Wedn. 7 Sep In $\mathbb{R}$-VS $\mathbf{V}$, collection $\mathcal{E} = \{\mathbf{u}_1, \ldots, \mathbf{u}_7\}$ is ***linearly-indep*** if ....

....

....

In VS $\mathbf{V}$, collection $\mathcal{B} \subset \mathbf{V}$ is a ***basis*** if ....

[Imagine 3 blank lines].

....

Q3: Frid. 16 Sep May lightning ↯ strike this table! (I.e, please fill in.)

n	r_n	q_n	s_n	t_n
0	131	—	**1**	**0**
1	56		**0**	**1**
2				
3				
4				
5				
6				

Thus $1 = [$ .... $\cdot 131] + [$ .... $\cdot 56]$.

Q4: Wedn. 21 Sep i The map $\text{PLY}_3 \to \text{PLY}_3$ which sends $f \mapsto g$, where $g(x) := x \cdot f'(x+5)$, is: Circle best Linear Affine Neither

ii Consider a linear map $\mathsf{T}: \mathbb{R}^3 \to \mathbb{R}^2$. Let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the standard basis for $\mathbb{R}^3$, and let $\{\mathbf{v}_1, \mathbf{v}_2\}$ be the standard basis for $\mathbb{R}^2$. Suppose that $\mathsf{T}(\mathbf{e}_1) = 17\mathbf{v}_1 - 2\mathbf{v}_2$ and $\mathsf{T}(\mathbf{e}_2) = 6\mathbf{v}_2$ and $\mathsf{T}(\mathbf{e}_3) = -4\mathbf{v}_1 - 3\mathbf{v}_2$.

Then the matrix of T is: ....

Q5: Frid. 30 Sep Let R_θ be the std. rotation [by θ] matrix. With

$$\mathsf{U} := \frac{1}{2} \cdot \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix} \quad \text{and} \quad \mathsf{M} := \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix},$$

the product $[\mathsf{UM}]^{45} = \alpha \cdot \mathsf{R}_\theta$, with $\alpha =$ .... $\in \mathbb{R}_+$ and $\theta =$ .... $\in (-180°, 180°]$. [*Hint:* Don't multiply matrices. Geometrically, U and M represent what linear-trns?]

Q6: Mond. 3 Oct Inverse of $\begin{bmatrix} 1 & -6 & -7 \\ & -3 & \\ & & 1 \end{bmatrix}$ is

.

A 2×2 matrix E has $\mathsf{E}^2 = \mathbf{0}$ (the zero-matrix). Then E itself must be $\mathbf{0}$. Circle: *T* *F*

Q7: Tues. 4 Oct From $\text{PLY}_4(\mathbb{Z}_7)$, place vectors

$$\begin{aligned}
\mathbf{v}_1 &:= 1 - x + 2x^2 + 6x^3, \\
\mathbf{v}_2 &:= 5 + x^2, \\
\mathbf{v}_3 &:= 5x + 5x^2 + 5x^3, \\
\mathbf{v}_4 &:= x + x^2 + 8x^3, \\
\mathbf{v}_5 &:= 1 + x + 3x^2 + 3x^3.
\end{aligned}$$

as columns in a 4×5 matrix. It RREFs to

.

A **basis** for $\text{Spn}(\{\mathbf{v}_1, \ldots, \mathbf{v}_5\})$ is this list of polynomials [just use the $\mathbf{v}$-symbols]: ....

Motto: *"Divide by multiplying!"*, by the mult-inverse.

[*Hint:* Write for yourself, the mod-7 reciprocal table.]

Q8: Wedn. 5 Oct From $\text{PLY}_4(\mathbb{Z}_{11})$, place vectors

$$\mathbf{w}_1 := 1 + 7x^2 + 4x^3,$$
$$\mathbf{w}_2 := 6 - 2x^2 + 2x^3,$$
$$\mathbf{w}_3 := \text{-}1 + 2x + 8x^2 - x^3,$$
$$\mathbf{w}_4 := \text{-}4 + 3x + 5x^3,$$
$$\mathbf{w}_5 := 3 - 2x + 9x^2 - x^3.$$

as columns in a 4×5 matrix. It RREFs to

A **basis** for $\text{Spn}(\{\mathbf{w}_1, \dots, \mathbf{w}_5\})$ is this list of polynomials [just use the **w**-symbols]: ______.

Motto: *"Divide by multiplying!"*, by the mult-inverse.

[*Hint:* Write for yourself, the mod-11 reciprocal table.]

Q9: Mond. 17 Oct Let $\mathsf{B} := \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 3 & 6 & 0 & -3 & 0 \end{bmatrix}$. Then $\mathsf{R} :=$ *RREF*(B) is [show no work, here]

$\mathsf{R} = \left[\quad | \quad | \quad | \quad | \quad \right]$.

I For subspace $\mathbf{V} := \text{Nul}(\mathsf{L}_\mathsf{B})$, use back-substitution, and *scaling*, to produce an <u>*integer basis*</u>

$\mathbf{v}_1 := (\_\_, \_\_, \_\_, \_\_, \_\_)$ $\mathbf{v}_2 := (\_\_, \_\_, \_\_, \_\_, \_\_)$

$\mathbf{v}_3 := (\_\_, \_\_, \_\_, \_\_, \_\_)$. $\mathbf{v}_4 := (\_\_, \_\_, \_\_, \_\_, \_\_)$

[*Note:* **Only use as many as the dimension of V.**]

QA: Tues. 18 Oct Perm $\pi := [6, 7, 8, 1, 2, 3, 4, 5]$ has $\text{Sgn}(\pi) =$ **+1** **−1**.

b In each blank below, write either "there exist" or "for all", Circle one of the underlined scalar-pairs, and Circle a phrase.

Assertion $\text{Spn}(\mathbf{v}, \mathbf{w}) \supset \text{Spn}(\mathbf{x}, \mathbf{y})$ *means:*

"______ *scalars* <u>*a, b*</u> | <u>*c, d*</u> (*st.* | *we have that* | *and*)
______ *scalars* <u>*a, b*</u> | <u>*c, d*</u> (*st.* | *we have that*)
$a\mathbf{v} + b\mathbf{w} = c\mathbf{x} + d\mathbf{y}$."

QB: Wedn. 19 Oct Let $\mathscr{L}$ be this list of 8 symbols: $R, G, R, T, U, \alpha, \beta, \omega$.

In matrix-eqn $\begin{bmatrix} R & U & G \\ & A & \\ T & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \omega \end{bmatrix}$, Cramer's Rule writes x_3 as ratio $f(\mathscr{L}) \big/ q(\mathscr{L})$ of polynomials

$f(\mathscr{L}) =$ ______

and $q(\mathscr{L}) =$ ______.

QC: Tues. 25 Oct $\mu =$ ______ $\leqslant \nu =$ ______ are the eigenvals of $\mathsf{G} := \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$. Let $\mathsf{D} := \begin{bmatrix} \mu & 0 \\ 0 & \nu \end{bmatrix}$. Then $\mathsf{D} = \mathsf{U}^{-1}\mathsf{G}\mathsf{U}$ where the 2×2 <u>*integer*</u> matrix U is

$\mathsf{U} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$.

QD: Tues. 08 Nov **i** Suppose C and A are 3×3 matrices s.t $\text{Det}(\mathsf{C}) = \frac{1}{2}$ and $\text{Det}(\mathsf{A}) = 5$. Then $\text{Det}(\mathsf{C}^{-1}\mathsf{A}\mathsf{C}^{\mathsf{t}}\mathsf{A}^{\mathsf{t}}\mathsf{A}\mathsf{C}^{\mathsf{t}}) =$ ______.

ii The 3×3 elem-matrix whose lefthand action adds 8 times row-2 to row-1 is

QE: Wedn. 09 Nov Let $\mathbf{u} := (1, \text{-}2, 3)$ and $\mathbf{w} := (11, 1, 4)$. Then

$\|\mathbf{u}\| =$ ______ and $\langle \mathbf{u}, \mathbf{w} \rangle =$ ______. Thus

$\mathbf{p} := \text{Proj}_\mathbf{u}(\mathbf{w}) =$ ______ and

$\mathbf{r} := \text{Orth}_\mathbf{u}(\mathbf{w}) =$ ______.

[*Hint:* $\text{Proj}_\mathbf{u}(\mathbf{p})$ should equal $\mathbf{p}$. DYCheck $\mathbf{r} \perp \mathbf{p}$?]

QBonus: In $\mathbb{C}^3$, let $\mathbf{v} := (2i, \text{-}1, 1)$. and $\mathbf{W} := \text{Spn}(\mathbf{v})$. Solve equation $\langle \mathbf{v}, \mathbf{u} \rangle = 0$ for $\mathbf{u}$, to get a *basis*

$\mathbf{u}_1 =$ ______ and $\mathbf{u}_2 =$ ______ for $\mathbf{W}^\perp$.

Gram-Schmidtify to obtain an orthogonal basis

$\mathbf{y}_1 =$ ______ and $\mathbf{y}_2 =$ ______.

[DYCheck that $\mathbf{y}_1 \perp \mathbf{v}$, $\mathbf{y}_2 \perp \mathbf{v}$, $\mathbf{y}_1 \perp \mathbf{y}_2$? –with both $\mathbf{y}_i \neq \mathbf{0}$.]

QF: Tue. 29 Nov Line $y = \Big[[\quad\quad\quad]x\Big] + \underline{\quad\quad\quad}$ is the least-squares best-fit to data pts $\Big\{(-2,0), (-1,0), (1,0), (2,1)\Big\}$.

QG: Wed. 30 Nov Fitting to data pts $\Big\{(-2,0), (-1,0), (1,0), (2,3)\Big\}$, parabola
$y = \underline{\quad\quad\quad} + \Big[[\quad\quad\quad]x\Big] + \Big[[\quad\quad\quad]x^2\Big]$
is the least-squares best-fit.

QH: Fri. 02 Dec ? *Solve <u>several of</u> of the World's Problems.*