Lina
MAS4105

Quizzes Q

29Sep2010

Q1: Wedn. 1 Sep Write $\text{Gcd}(266, 209)$ as a lin-combination, using

n	r_n	q_n	s_n	t_n
0	266	—	**1**	**0**
1	209		**0**	**1**
2				
3				
4				
5				

So ______ = __________ $\cdot\, 266 +$ __________ $\cdot\, 209$.

Q2: Fri. 3 Sep Since $M := 211$ is prime, ring $\mathbb{Z}_{211}$ is a field. So the mod-M reciprocal of 9 is $R :=$ __________ $\in [0\,..\,M)$.
[IOWords, $9 \cdot R \equiv_{211} 1$ and $R \in [0\,..\,M)$.]

Q3: Fri. 17Sep Fix a field $\mathbb{F}$. A map $g\colon(\mathbf{H}, +, \mathbf{0_H}) \to (\mathbf{E}, +, \mathbf{0_E})$ between two $\mathbb{F}$-VSes is $\mathbb{F}$-***linear*** if [Remember quantification]:

[Imagine 3 blank lines].

Using set-builder notation, the ***kernel*** of g is:
$\text{Ker}(g) =$ ______________________________.

Q4: Tue. 21Sep Let $\mathsf{M} := \begin{bmatrix} 0 & 0 & 1 & 1 & 3 \\ 1 & -4 & 5 & -1 & -32 \\ 0 & 0 & 5 & 4 & 6 \end{bmatrix}$. Working over field $\mathbb{Z}_5$, matrix $RREF(\mathsf{M})$ equals [write entries in [-2 .. 2] please]

$$\begin{bmatrix} \ & \ & \ & \ & \ \\ \ & \ & \ & \ & \ \\ \ & \ & \ & \ & \ \end{bmatrix}$$

Q5: Fri. 24Sep

a) The 3×3 elem-matrix whose lefthand action adds 2 times row-1 to row-3 is $\begin{bmatrix} \ & \ & \ \\ \ & \ & \ \\ \ & \ & \ \end{bmatrix}$.

b) Fix a field $\mathbf{F}$. A map $f\colon(\mathbf{E}, +, \mathbf{0_E}) \to (\mathbf{H}, +, \mathbf{0_H})$ between two $\mathbf{F}$-VSes is $\mathbf{F}$-***linear*** if [Remember Qfn!]:

[Imagine 3 blank lines].

Q6: Mond. 27Sep Over field $\mathbb{Z}_3$, polynomials $\mathbf{f}_1(x) := [x+1]^3$, $\mathbf{f}_2(x) := x^3 + x + 1$, $\mathbf{f}_3(x) := [x-1]\cdot[x+2]$, $\mathbf{f}_4(x) := x^2$, $\mathbf{f}_5(x) := x^3 + 2$. satisfy $\mathbf{f}_5 = \sum_{j=1}^{4} \alpha_j \mathbf{f}_j$, where coeffs
$\alpha_1 =$ ____, $\alpha_2 =$ ____, $\alpha_3 =$ ____, $\alpha_4 =$ ____, lie in $\mathbb{Z}_3$.

Q7: Wed. 29Sep Poly $f(x) := \sum_{n=0}^{3} \alpha_n x^n$ satisfies $f(7) = f(9) = f(-3) = 0$ and $f(4) = 5$. Then $\alpha_3 =$ ______ and $\alpha_0 =$ ______.
[Write each α as $\dfrac{a \cdot b \cdots}{p \cdot q \cdots}$, a ratio of *integer*-products.]

Q8: Fri. 8Oct Let $\mathsf{T}\colon\mathbb{R}^2 \to \mathbb{R}^2$ by $\mathsf{T}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) := \begin{bmatrix} 3x - y \\ 2x + 6y \end{bmatrix}$. W.r.t ordered-basis $\mathcal{B} := \left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right)$, let $\mathsf{M} := [\![\mathsf{T}]\!]_{\mathcal{B}}^{\mathcal{B}}$. Then $\mathsf{M} = \mathsf{R}\mathsf{T}\mathsf{R}^{-1}$,

where $\mathsf{R} = \begin{bmatrix} \ & \ \\ \ & \ \end{bmatrix}$, $\mathsf{M} = \begin{bmatrix} \ & \ \\ \ & \ \end{bmatrix}$.

What I had intended was:
Let $\mathsf{T}\colon\mathbb{R}^2 \to \mathbb{R}^2$ *by* $\mathsf{T}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) := \begin{bmatrix} -x + 15y \\ -2x + 10y \end{bmatrix}$. *W.r.t ordered-basis* $\mathcal{B} := \left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right)$, *let* $\mathsf{M} := [\![\mathsf{T}]\!]_{\mathcal{B}}^{\mathcal{B}}$. *Then* $\mathsf{M} = \mathsf{R}\mathsf{T}\mathsf{R}^{-1}$,

where $\mathsf{R} = \begin{bmatrix} \ & \ \\ \ & \ \end{bmatrix}$, $\mathsf{M} = \begin{bmatrix} \ & \ \\ \ & \ \end{bmatrix}$.

EC: Mon. 18Oct Let R_θ be the std. rotation [by θ] matrix. With

$$\mathsf{C} := \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix} \quad\text{and}\quad \mathsf{B} := \begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix},$$

the product $[\mathsf{CB}]^{35} = \alpha \cdot \mathsf{R}_\theta$, with $\alpha =$ __________ $\in \mathbb{R}_+$ and $\theta =$ __________ $\in (-180°, 180°]$. [*Hint:* Don't multiply matrices. Geometrically, C and B represent what linear-trns?]

Q9: Mon. 25Oct Perm $\pi := [6, 7, 8, 1, 2, 3, 4, 5]$ has $\text{Sgn}(\pi) = +\mathbf{1}\ -\mathbf{1}$.

Let $\mathsf{M}(x) := \begin{bmatrix} 10 & 7x^4 - 8 & 3x - 2 \\ 2x - 8 & 9x - 2 & 5 \\ x^3 + 2 & x^5 - 2 & 8x \end{bmatrix}$.

The high-order term of polynomial $\text{Det}(\mathsf{M}(x))$ is Cx^N, where $C =$ __________ and $N =$ __________.

Q10: Tues. 26Oct Apply Cramer's Rule to give a formula for $x_2 =$ ______

ITOf B, C, D, E, z, y, where $\begin{bmatrix} B & C \\ D & E \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} z \\ 1+y \end{bmatrix}$.

Q11: Mond. 08Nov

$\mu =$ ______ $\leqslant \nu =$ ______

are the eigenvals of $\mathsf{G} := \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$. Let $\mathsf{D} := \begin{bmatrix} \mu & 0 \\ 0 & \nu \end{bmatrix}$. Then $\mathsf{D} = \mathsf{U}^{-1}\mathsf{G}\mathsf{U}$ where the 2×2 *integer* matrix U is

$$\mathsf{U} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}.$$

ED: Wedn. 17Nov The point $P := (-3, 1)$, in the plane, has orthogonal projection $Q := ($ ______ , ______ $)$ on $\mathbb{L}$, the line $y = -5 + 2x$. [*Check* that $Q \in \mathbb{L}$ and $[P - Q] \perp \mathbb{L}$.]

E12: Tue. 23Nov On **V**, the VS of 2-topped real polynomials, put IP $\langle f, g\rangle := \int_0^1 [f \cdot g]$. The two polys 1 and x form a basis for **V**. Define a linear-fnc'al $L\colon \mathbf{V} \to \mathbb{R}$ by $L(1) := 3$ and $L(x) := 5$. The general theory tells us there is a (unique) poly $h \in \mathbf{V}$ st. $\langle h, \cdot\rangle = L(\cdot)$. So $h(x) = S + Tx$ for numbers $S=$ ______ and $T=$ ______.

E13: Mon. 29Nov Line $y = \big[[$ ______ $]x\big] +$ ______ is the least-squares best-fit to data pts $\{(-2, 0), (-1, 0), (1, 0), (2, 1)\}$.

§A Potential quiz problems

Some of these may someday appear on quizzes/exams; naturally, with different data. *Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.*

Note: *A few ask about material we have not yet covered.*

L1 In $\mathbb{R}^3$, the point $P := (2, -1, 3)$ has orthogonal projection $\text{Proj}(P) = ($ ______ , ______ , ______ $)$ on the line passing through $(2, 2, 4)$ and the origin.

L2 Let $\mathbf{u} := (1 + 2\boldsymbol{i}, 3 - \boldsymbol{i}, 1)$ and $\mathbf{w} := (1, 1 + \boldsymbol{i}, 2 - 3\boldsymbol{i})$. Then $\|\mathbf{u}\| =$ ______ and $\langle \mathbf{u}, \mathbf{w}\rangle =$ ______. Thus $\mathbf{p} := \text{Proj}_{\mathbf{u}}(\mathbf{w}) =$ ______ and $\mathbf{r} := \text{Orth}_{\mathbf{u}}(\mathbf{w}) =$ ______.

[*Hint:* $\text{Proj}_{\mathbf{u}}(\mathbf{p})$ should equal $\mathbf{p}$. DYCheck $\mathbf{r} \perp \mathbf{p}$?]

L3 The point $P := (5, -1)$, in the plane, has orthogonal proj. $\text{Proj}(P) = ($ ______ , ______ $)$ on the line $y = 1 + 3x$.

e1 $\mu =$ ______ $\leqslant \nu =$ ______

are the eigenvals of $\mathsf{G} := \begin{bmatrix} -1 & 8 \\ 2 & -1 \end{bmatrix}$. Let $\mathsf{D} := \begin{bmatrix} \mu & 0 \\ 0 & \nu \end{bmatrix}$. Then $\mathsf{D} = \mathsf{U}^{-1}\mathsf{G}\mathsf{U}$ where the 2×2 *integer* matrix U is

$$\mathsf{U} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}.$$

e2 $\mathsf{M} := \begin{bmatrix} -5 & 3 & 18 \\ -2 & 0 & 12 \\ -4 & 4 & 7 \end{bmatrix}$ has three real eigenvalues,

$\alpha =$ ______ $\leqslant \beta =$ ______ $\leqslant \gamma =$ ______.

Hence $\begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix} = \mathsf{U}^{-1}\mathsf{M}\mathsf{U}$, where

$$\mathsf{U} = \begin{bmatrix} \quad & \quad & \quad \end{bmatrix}.$$

CB1 With C the change-of-basis matrix <u>from</u> $\mathcal{E} := (1, x, x^2)$ <u>to</u> $\mathcal{B} := (3x + 5x^2, x + 2x^2, 1)$, then C^{-1} equals

$$\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}, \quad \mathsf{C} = \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}.$$

CR Matrix $\mathsf{M} = \begin{bmatrix} \mathsf{A} & \mathsf{B} \\ \mathsf{C} & \mathsf{D} \end{bmatrix}$, where A and D are 5×5 and 7×7, resp. Suppose C is the 7×5 zero-matrix. <u>Prove</u> that $\text{Det}(\mathsf{M}) = \text{Det}(\mathsf{A}) \cdot \text{Det}(\mathsf{D})$. [*Hint:* A good picture helps.]

VS1 Below, **V**, **W**, **E** are real vectorspaces.

[a] A map $f\colon \mathbf{V}\times\mathbf{W}\to\mathbf{E}$ is ***bilinear*** if...

[b] A map $\langle\cdot,\cdot\rangle$ from $\mathbf{V}\times\mathbf{V}\to\mathbb{R}$ is an ***inner product*** if...

[R0] Let $\mathsf{M} := \begin{bmatrix} 1 & 5 & -1 \\ 3 & 0 & -6 \\ -2 & -1 & 1 \end{bmatrix}$. Viewing M as a *rational* matrix, compute:

A basis $\mathcal{R}$ for RowNullspace(M).

A basis $\mathcal{C}$ for ColSpan(M). Now write each M-col as an *explicit* linear-comb. of vectors in your $\mathcal{C}$.

Finally, interpret M as a $\mathbb{Z}_7$–matrix, and answer the same three questions.

[p_0] OYOP, write out the following sentences, and complete them to give the correct definitions. Be specific with phrases "each", "every", "all", "there exists", etc.. *Avoid* the word "any". Use "**there exists**" in preference to "some".

*A (possibly infinite) set $\mathcal{S} \subset \mathbf{V}$ of vectors is **linearly dependent** IFF ...*

*Vector $\mathbf{w}$ is in the **span** of $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_K\}$ IFF ...*

*A $K\times N$ matrix U is in **reduced row echelon form** IFF ...*

*The **nullspace** of $\mathsf{T}\colon\mathbf{V}\to\mathbf{W}$ is the set of all ...*

[ℓ] Poly $h(x) := \sum_{n=0}^{2} V_n x^n$ satisfies $h(1)=4$, $h(2)=9$, $h(-1)=6$. Then $V_0=$ ______, $V_1=$ ______, $V_2=$ ______.

[S] A system of 3 linear equations in unknowns $x_1, \ldots, x_5$ reduces to the augmented matrix

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 1 & 12 \\ 0 & 0 & 1 & 0 & -8 & 34 \\ 0 & 0 & 0 & 1 & 5 & -56 \end{array}\right],$$

which is in RREF. Please (circle) each *pivot entry.*

OYOP, describe the *general solution* in this form,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \alpha \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \beta \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \gamma \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \ldots$$

where each $\alpha, \beta, \gamma, \delta, \ldots$ is a free variable (either x_1 or... or x_5), and each column vector has specific numbers in it. Dim(*SolnFlat*) = ______.

[m_1] Matrix-product $\left[\begin{smallmatrix} b \\ c \end{smallmatrix}\right] \cdot \left[\begin{smallmatrix} x & y \end{smallmatrix}\right]=$ ______.

[m_2] The matrix-product $\begin{bmatrix} 2 & 1 & 4 \\ 0 & -1 & 1 \\ -2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \sqrt{2} & 0 & \sqrt{2} \\ 0 & -1 & 5 \end{bmatrix}$ equals

______.

[m_3] Find 2×2 matrices E and G with $\mathsf{E}^2 - \mathsf{G}^2$ **unequal** to $[\mathsf{E}-\mathsf{G}]\cdot[\mathsf{E}+\mathsf{G}]$: $\mathsf{E} :=$ ______. $\mathsf{G} :=$ ______.

$\mathsf{E}^2 - \mathsf{G}^2 =$ ______. $[\mathsf{E}-\mathsf{G}][\mathsf{E}+\mathsf{G}] =$ ______.

[m_4] A 2×2 matrix E has $\mathsf{E}^2 = \mathbf{0}$ (the zero-matrix). Then E itself must be $\mathbf{0}$. (Circle): *T* *F*

[m_5] $\mathsf{M} := \begin{bmatrix} 70 & 7 \\ 1 & 2 \end{bmatrix}$. Compute M^{-1} over these three fields. [Write your $\mathbb{Z}_p$ answers using symmetric residues.]

Over $\mathbb{Z}_5$: $\mathsf{M}^{-1}=$ ______. Over $\mathbb{Z}_7$: $\mathsf{M}^{-1}=$ ______.

Over $\mathbb{Q}$: $\mathsf{M}^{-1}=$ ______.

[m_6] Mats $\mathsf{U}= \begin{bmatrix} & \\ & \end{bmatrix}$, $\mathsf{V}= \begin{bmatrix} & \\ & \end{bmatrix}$ are $\mathbb{R}$-matrices such that $\mathsf{U}^2 \neq \mathsf{V}^2$, yet $\mathsf{U}^3 = \mathsf{V}^3$.

[m_7] Consider these two matrices:

$$\mathsf{R} := \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \quad\text{and}\quad \mathsf{A} := \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}.$$

Product matrix

$$[RA]^{40} = \begin{bmatrix} & \\ & \end{bmatrix}.$$

[*Hint:* You don't need multiply matrices. Geometrically, what motion do these matrices represent?]

[m_8] Consider these two matrices:

$$\mathsf{C} := \tfrac{1}{2}\cdot\begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix} \quad\text{and}\quad \mathsf{B} := \tfrac{1}{2}\cdot\begin{bmatrix} \sqrt{2} & \sqrt{2} \\ -\sqrt{2} & \sqrt{2} \end{bmatrix}.$$

Determine the matrix $[\mathsf{CB}]^{44}=$ ______.

[*Hint:* You don't need multiply matrices. Geometrically, what motion do these matrices represent.]

m_9 Consider a linear map $\mathsf{T}:\mathbb{R}^3\to\mathbb{R}^2$. Let $\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}$ be the standard basis for $\mathbb{R}^3$, and let $\{\mathbf{v}_1,\mathbf{v}_2\}$ be the standard basis for $\mathbb{R}^2$. Suppose that $\mathsf{T}(\mathbf{e}_1) = 17\mathbf{v}_1 - 2\mathbf{v}_2$ and $\mathsf{T}(\mathbf{e}_2) = 6\mathbf{v}_2$ and $\mathsf{T}(\mathbf{e}_3) = \text{-}4\mathbf{v}_1 - 3\mathbf{v}_2$.

Then the matrix of T is: ________________ .

m_{10} Shear the plane *vertically*, sending $\mathbf{e}_1$ to $\mathbf{e}_1 + 3\mathbf{e}_2$, followed by the *horizontal* shear which sends $\mathbf{e}_2$ to $-2\mathbf{e}_1+\mathbf{e}_2$. Let S be the $2{\times}2$ matrix whose effect is the preceding composition of shears.

Then $\mathsf{S} = \begin{bmatrix} & \\ & \end{bmatrix}$.

R1 Let $\mathsf{M} := \begin{bmatrix} 0 & 0 & 5 & 3 & 0 \\ 5 & 3 & 5 & 2 & 0 \\ 5 & 3 & 1 & 1 & 3 \end{bmatrix}$. Working over field $\mathbb{Z}_{13}$, matrix $\mathit{RREF}(\mathsf{M})$ equals [write entries in [-6 .. 6] please]

$$\begin{bmatrix} & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

R2 Let $\mathbf{v}_1 := \begin{bmatrix} 0 \\ 1 \\ \text{-}1 \end{bmatrix}$, $\mathbf{v}_2 := \begin{bmatrix} 2 \\ 1 \\ \text{-}2 \end{bmatrix}$, $\mathbf{v}_3 := \begin{bmatrix} 4 \\ Y \\ 3 \end{bmatrix}$. Our $\mathbf{v}_3$ is in $\operatorname{Spn}(\mathbf{v}_1,\mathbf{v}_2)$ when number $Y=$ ________ . And then, $\mathbf{v}_3 = \alpha\mathbf{v}_1 + \beta\mathbf{v}_2$, where $\alpha=$ ______ and $\beta=$ ______ .

R3 Let $\mathbf{v}_1 := \begin{bmatrix} 0 \\ 1 \\ \text{-}1 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 := \begin{bmatrix} 2 \\ 1 \\ \text{-}2 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 := \begin{bmatrix} 4 \\ W \\ 3 \\ Y \end{bmatrix}$, So $\mathbf{v}_3 \in \operatorname{Spn}(\mathbf{v}_1,\mathbf{v}_2)$ when $W=$ ________ & $Y=$ ________ .

And $\mathbf{v}_3 = \alpha\mathbf{v}_1 + \beta\mathbf{v}_2$, where $\alpha=$ ______ and $\beta=$ ______ .

V_1 Inverse of $\begin{bmatrix} 1 & \text{-}6 & -7 \\ & \text{-}3 & \\ & & 1 \end{bmatrix}$ is

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}.$$

V_2 Inverse of $\begin{bmatrix} 6 & \text{-}7 & \text{-}7 \\ 3 & \text{-}4 & \text{-}2 \\ 1 & \text{-}3 & 4 \end{bmatrix}$ is

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}.$$

V_3 Over $\mathbb{Q}$, the inverse of $\mathsf{E} := \begin{bmatrix} 1 & x & z \\ & 1 & y \\ & & 1 \end{bmatrix}$ is

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}.$$

E The $3{\times}3$ elem-matrix whose lefthand action adds 8 times row-2 to row-1 is $\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$.

d1 Suppose C and A are $3{\times}3$ matrices s.t $\operatorname{Det}(\mathsf{C}) = \frac{1}{2}$ and $\operatorname{Det}(\mathsf{A}) = 5$. Then $\operatorname{Det}(\mathsf{C}^{-1}\mathsf{A}\mathsf{C}^{\mathsf{t}}\,\mathsf{A}^{\mathsf{t}}\,\mathsf{A}\mathsf{C}^{\mathsf{t}})=$ ________________ .

Definitions, and their application

Use $\mathbb{F}$ for a general field, and $\mathbf{V}$ is an $\mathbb{F}$-VS.

p_1 A collection $\{\mathbf{u}_1,\ldots,\mathbf{u}_{17}\} \subset \mathbf{V}$ is ***linearly independent*** **(*over* $\mathbb{F}$)** if: (Qfn!) ________________

[Imagine 3 blank lines]. ________________________________ .

p_2 For a subset $S \subset \mathbf{V}$ write $\operatorname{Spn}(S)$ using set-builder notation. [Note: S can be infinite.] ________________

________________________________ .

p_3 Let $c_1,\ldots,c_5$ be the columns of $\mathsf{M} := \begin{bmatrix} 27 & -108 & 5 & 2 & 177 \\ 5 & -20 & 29 & 17 & 14 \\ 2 & -8 & 17 & 10 & 2 \end{bmatrix}$. Now $\mathit{RREF}(\mathsf{M})$ equals $\mathsf{R} := \begin{bmatrix} 1 & \text{-}4 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & \text{-}6 \\ 0 & 0 & 0 & 1 & 9 \end{bmatrix}$. Using the <u>*minimum*</u> of columns, write $\begin{bmatrix} 177 \\ 14 \\ 2 \end{bmatrix} =$ ______ $c_1 +$ ______ $c_2 +$ ______ $c_3 +$ ______ c_4.

p_4 For $\mathbf{w}, \mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_{35} \in \mathbf{V}$, saying that $\mathbf{w}$ is an ***affine combination*** (*over* $\mathbb{F}$) of $\mathbf{u}_1, \ldots, \mathbf{u}_{35}$ means (Qfn!) [Imagine 3 blank lines].

Aff Glued to a massless plate is a 10 lb weight at the origin, a 15 lb weight at the point $(3, -1)$, and 5 lb at point (______ , ______), thus putting the center-of-mass of the weighted-plate at $(2, 1)$.

p_5 A subset $S \subset \mathbf{V}$ is **"a *flat*"** if: ________

[Imagine 3 blank lines].

LI: *Here*, let *AT* mean "Always True", *AF* mean "Always False" and *Nei* mean "Neither always true nor always false". Below, $\mathbf{v}, \mathbf{w}, \mathbf{x}$ repr. *distinct, non-zero* vectors in $\mathbb{R}^4$, a $\mathbb{R}$-VS. Please circle the correct response:

y1 If $\mathbf{x} \notin \mathrm{Spn}\{\mathbf{v}, \mathbf{w}\}$ then $\{\mathbf{v}, \mathbf{w}, \mathbf{x}\}$ is linearly independent.

AT *AF* *Nei*

y2 Collection $\{\mathbf{0}, \mathbf{x}\}$ is linearly-independent.

AT *AF* *Nei*

y3 $\mathrm{Spn}\{\mathbf{v}, \mathbf{w}, \mathbf{x}, \mathbf{v} + 2\mathbf{w} + 3\mathbf{x}\}$ is all of $\mathbb{R}^4$.

AT *AF* *Nei*

y4 If none of $\mathbf{v}, \mathbf{w}, \mathbf{x}$ is a multiple of the other vectors, then $\{\mathbf{v}, \mathbf{w}, \mathbf{x}\}$ is linearly independent.

AT *AF* *Nei*