DfyQ MAP2302 **Quizzes Q** Wednesday 15Jan2020

Number Sets. Expression $k \in \mathbb{N}$ [read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$"] means that k is a natural number; a ***natnum***. Expression $\mathbb{N} \ni k$ [read as "$\mathbb{N}$ ***owns*** k"] is a synonym for $k \in \mathbb{N}$.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \dots\}$.

$\mathbb{Z}$ = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive rationals and $\mathbb{Q}_-$ for the negative rationals.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

For $\omega \in \mathbb{C}$, let "$\omega > 5$" mean "ω *is* ***real*** and $\omega > 5$". [Use the same convention for $\geqslant, <, \leqslant$, and also if 5 is replaced by any real number.]

An "***interval of integers***" $[b\,..\,c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e}\,..\,2\pi] = \{3, 4, 5, 6\} = [3\,..\,6] = (2\,..\,6]$. We allow b and c to be $\pm\infty$; so $(-\infty\,..\,-1]$ is $\mathbb{Z}_-$. And $[-\infty\,..\,-1]$, is $\{-\infty\} \cup \mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2i| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'. RoC: '*Radius of Convergence*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand Side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', but pt: '*point*', as in "a fixed-pt of a map".

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** fnc, defined for $x>0$, is $\log(x) := \int_1^x \frac{\mathrm{d}v}{v}$. Its inverse-fnc is $\exp()$. For $x>0$, then, $\exp(\log(x)) = x = \mathrm{e}^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(\mathrm{e}^t)$.

PolyExp: '*Polynomial-times-exponential*'; e.g, $[3 + t^2]\cdot\mathrm{e}^{4t}$. PolyExp-sum: '*Sum of polyexps*'. E.g, $f(t) := 3t\mathrm{e}^{2t} + [t^2]\cdot\mathrm{e}^t$ is a polyexp-sum.

Prefix nt- means '*non-trivial*'. E.g "a *nt*-soln to $f' = 5f$ is $f(t) := \mathrm{e}^{5t}$; a *trivial* soln is $f \equiv 0$."

Phrases. WLOG: '*Without loss of generality*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. Use iff: '*if and only if*'.

IST: '*It Suffices to*' as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. inter alia: '*among other things*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Factorial. Def: $n! := n\cdot[n-1]\cdot[n-2]\cdots2\cdot1$; so $0! = 1$.

Rising Fctrl: $[\![x \uparrow K]\!] := x\cdot[x+1]\cdot[x+2]\cdots[x+[K-1]]$,

Falling Fctrl: $[\![x \downarrow K]\!] := x\cdot[x-1]\cdot[x-2]\cdots[x-[K-1]]$,

for natnum K and $x \in \mathbb{C}$. E.g, $[\![K \downarrow K]\!] = K! = [\![1 \uparrow K]\!]$.

N.B: For $n \in \mathbb{N}$: If $K > n$ then $[\![n \downarrow K]\!] = 0$.

Note $[\![x \uparrow K]\!] = [\![x + [K-1] \downarrow K]\!]$.

DfyQ [2020g] quizzes so far...

Q1: Fri. 24 Jan $\left[\left[\sqrt[3]{2}\right]^{\sqrt{2}}\right]^{\sqrt{8}} =$ ________. $\log_8(4) =$ ________.

Q2: Fri. 24 Jan U.F $y = y(t)$ satisfies $y'' + 3y' + 2y = 0$, with $y(0) = 2$ and $y'(0) = 11$. So $y(t) = A\mathrm{e}^{\alpha t} + B\mathrm{e}^{\beta t}$ where

$\alpha=$ ________, $\beta=$ ________, $A=$ ________, $B=$ ________.

Q3: Mon. 27 Jan Blanks$\in\mathbb{R}$. So $\frac{1}{2+3\boldsymbol{i}} =$ ________ $+\boldsymbol{i}\cdot\left[\text{________}\right]$.

Thus $\operatorname{Im}\left(\frac{5-\boldsymbol{i}}{2+3\boldsymbol{i}}\right) =$ ________.

By the way, $|5 - 3\boldsymbol{i}| =$ ________.

Q4: Wed. 29 Jan *A* particular polynomial $p=p(t)$ satisfying

$$*: \quad p' + 2p = 6t^2 + 8t + 1$$

is $p(t) =$ ________ $\cdot t^2 +$ ________ $\cdot t +$ ________.

The general soln has form $y_\alpha(t) = \alpha\mathrm{e}^{Mt} + p(t)$, where $M=$ ________. [Put correct numbers in the four blanks.]

Q5: Fri. 31 Jan For CCLDOp $\mathsf{L} := \mathbf{D}^3 - 3\mathbf{D} + 2\mathbf{I}$ and thrice diff'able fnc f, note $\mathsf{L}(f\cdot\mathrm{e}^{2t}) = \mathsf{V}(f)\cdot\mathrm{e}^{2t}$, where CCLDOp V is

$\mathsf{V} =$ ________ $\mathbf{D}^3 +$ ________ $\mathbf{D}^2 +$ ________ $\mathbf{D} +$ ________ $\mathbf{I}$.

[Put the correct number in *each* of the four blanks; zero, one, fractions, and negative numbers are allowed.]

Q6: Mon. 03 Feb For $t>0$, fnc $y_\alpha(t) :=$ ________

is the gen.soln to $ty' + 2y = t^4$. [*Hint:* FOLDE. The coefficient and target fncs are $C(t) := \frac{2}{t}$ and $G(t) := t^3$.]

Q7: Fri. 07 Feb For $t>0$, fnc $y_\alpha(t) :=$ ________

is the gen.soln to $y' + \left[\frac{2}{t}\cdot y\right] = t^3$. [*Hint:* FOLDE. The coefficient and target fncs are $C(t) := \frac{2}{t}$ and $G(t) := t^3$.]

Q8: Wed. 19 Feb DE $\left[\mathcal{N}(x,y)\cdot\frac{\mathrm{d}y}{\mathrm{d}x}\right] + \mathcal{M}(x,y) = 0$ is *exact*, where

$\mathcal{N}(x,y) := \left[x^2 - 7\right]$ and $\mathcal{M}(x,y) := 2xy + 3\mathrm{e}^{3x}$.

Its solns $y=y(x)$ satisfy $\mathbf{F}(x, y(x)) = \text{Const}$, where

$\mathbf{F}(x,y) =$ ________.

Q9: Fri. 21 Feb DE $\left[[2x+8]y\cdot\frac{\mathrm{d}y}{\mathrm{d}x}\right] + 4y^2 = 0$ is <u>not</u>, alas, *exact*. Happily, multiplying both sides by (non-constant) fnc $V(y) =$ ________

gives a *new* DE which *is* exact.

Solving the exact-DE, every (non-zero) soln $y=y(x)$ satisfies $F(x, y(x)) = \alpha$, for some constant α, where

$F(x,y) =$ ________.

QA: Fri. 28 Feb A *minimum* requirement for an LOR (letter-of-recommendation) from Prof. K is <u>two</u> courses. Circle:

Yes **True** **Darn tootin'!**

QB: Mon. 09 Mar With $\mathbf{1}()$ the constant-1 fnc and $F(x) := \sin(5x)$, then, convolution

$\left[\mathbf{1} \circledast F\right](x) =$ ________.

QC: ?? Wed. 11 Mar Solve some of the World's Problems.