Number Sets. An expression such as $k \in \mathbb{N}$ (read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$") means that k is a natural number; a ***natnum***.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \dots\}$.

$\mathbb{Z}$ = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive ***ratnums*** and $\mathbb{Q}_-$ for the negative ratnums.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

For $\omega \in \mathbb{C}$, let "$\omega > 5$" mean "ω *is* ***real*** and $\omega > 5$". [Use the same convention for $\geqslant, <, \leqslant$, and also if 5 is replaced by any real number.]

An "***interval of integers***" $[b \,..\, c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e} \,..\, 2\pi] = \{3, 4, 5, 6\} = [3 \,..\, 6] = (2 \,..\, 6]$. We allow b and c to be $\pm\infty$; so $(-\infty \,..\, -1]$ is $\mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2i| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand Side*' of an eqn or inequality. LhS: '*left-hand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', *but* pt: '*point*', as in "a fixed-pt of a map".

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** fnc, defined for $x>0$, is $\log(x) := \int_1^x \frac{dv}{v}$. Its inverse-fnc is $\exp()$. For $x>0$, then, $\exp(\log(x)) = x = \mathrm{e}^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(\mathrm{e}^t)$. PolyExp: '*Polynomial-times-exponential*'. E.g, $F(t) := [3 + t^2]\cdot\mathrm{e}^{4t}$ is a polyExp.

Prefix nt- means '*non-trivial*'. E.g "a *nt*-soln to $f' = 5f$ is $f(t) := \mathrm{e}^{5t}$; a *trivial* soln is $f \equiv 0$."

Phrases. WLOG: '*Without loss of generality*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. Use iff: '*if and only if*'.

IST: '*It Suffices to*' as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Factorial. Def: $n! := n\cdot[n-1]\cdot[n-2]\cdots 2\cdot 1$; so $0! = 1$.

Rising Fctrl: $[\![x \uparrow K]\!] := x\cdot[x+1]\cdot[x+2]\cdots[x+[K-1]]$,

Falling Fctrl: $[\![x \downarrow K]\!] := x\cdot[x-1]\cdot[x-2]\cdots[x-[K-1]]$,

for natnum K and $x \in \mathbb{C}$. E.g, $[\![K \downarrow K]\!] = K! = [\![1 \uparrow K]\!]$.

N.B: For $n \in \mathbb{Z}$: If $K > n$ then $[\![n \downarrow K]\!] = 0$.

Note $[\![x \uparrow K]\!] = [\![x + [K-1] \downarrow K]\!]$.

Sample questions

Q: Wed. 27Sep The solutions to $3x^2 = 2 - 2x$ are $x =$ ______.

Q: Wed. 27Sep $\left[\left[\sqrt[3]{2}\right]^{\sqrt{2}}\right]^{\sqrt{8}} =$ ______. $\log_8(4) =$ ______.

DfyQ [2018t] quizzes so far...

Q1: Wed. 29 Aug $\left[\sqrt{3}^{\sqrt{8}}\right]^{\sqrt{2}} =$ ______. $\log_{64}(16) =$ ______.

Q2: Fri. 31 Aug U.F $y = y(t)$ satisfies $y'' + 3y' + 2y = 0$, with $y(0) = 2$ and $y'(0) = 11$. So $y(t) = A\mathrm{e}^{\alpha t} + B\mathrm{e}^{\beta t}$ where

$\alpha =$ ______, $\beta =$ ______, $A =$ ______, $B =$ ______.

Q3: Wed. 05 Sep Blanks$\in\mathbb{R}$. So $\frac{1}{2+3i} =$ ______ $+ i \cdot$ [______].

Thus $\mathrm{Im}\left(\frac{5 - i}{2 + 3i}\right) =$ ______.

By the way, $|5-3\boldsymbol{i}| =$ ____.

Q4: Wed. 12 Sep Value $\mathrm{Im}(\frac{1}{4-3\boldsymbol{i}}) =$ ____.

Suppose $[C + D\boldsymbol{i}]^2 = -4\boldsymbol{i}$, where $C,D \in \mathbb{R}$. Then $C =$ ____ and $D =$ ____.

Q5: Fri. 14 Sep For $t>0$, fnc $y_\alpha(t) :=$ ____

is the gen.soln to $y' + \left[\frac{2}{t} \cdot y\right] = t^3$. [*Hint:* FOLDE. The coefficient and target fncs are $C(t) := \frac{2}{t}$ and $G(t) := t^3$.]

Q6: Mon. 17 Sep DE $[\mathcal{N}(x,y) \cdot \frac{\mathrm{d}y}{\mathrm{d}x}] + \mathcal{M}(x,y) = 0$ is *exact*, where

$\mathcal{N}(x,y) := [x^2-7]$ and $\mathcal{M}(x,y) := 2xy + 3\mathrm{e}^{3x}$.

Its solns $y=y(x)$ satisfy $\mathbf{F}(x, y(x)) = \text{Const}$, where

$\mathbf{F}(x,y) =$ ____.

Q7: Fri. 21 Sep *A* particular soln $p=p(t)$ to

$$p' + 2p = 6t^2 + 8t + 1$$

is $p(t) =$ ____ $\cdot t^2 +$ ____ $\cdot t +$ ____.

The general soln is $y_\alpha(t) = \alpha\mathrm{e}^{Mt} + p(t)$, where $M=$ ____. [Put correct numbers in the four blanks.]

Q8: Mon. 24 Sep The DOp in $y' - 2y \stackrel{*}{=} [4t+3]\mathrm{e}^{3t}$ is $\mathsf{L} := \mathbf{D} - 2\mathbf{I}$.

It's *associated op* $\mathsf{V}=\mathsf{V}_{\mathsf{L},3}$ satisfies $\mathsf{L}(f\mathrm{e}^{3t}) = \mathsf{V}(f)\,\mathrm{e}^{3t}$, where $\mathsf{V}(f) =$ ____. Thus $y := G(t)\mathrm{e}^{3t}$ satisfies $(*)$, for *polynomial* $G(t) =$ ____.

Q9: Fri. 05 Oct **A** soln (use PolyExp) to

$y'' + y = [4t-2]\mathrm{e}^t$ is $y(t)=$ ____.

QA: Mon. 08 Oct For CCLDOp $\mathsf{L} := \mathbf{D}^3 - 3\mathbf{D} + 2\mathbf{I}$ and thrice diff'able fnc f, note $\mathsf{L}(f\cdot\mathrm{e}^{2t}) = \mathsf{V}(f)\cdot\mathrm{e}^{2t}$, where CCLDOp V is

$\mathsf{V} =$ ____ $\mathbf{D}^3 +$ ____ $\mathbf{D}^2 +$ ____ $\mathbf{D} +$ ____ $\mathbf{I}$.

[Put the correct number in *each* of the four blanks; zero, one, fractions, and negative numbers are allowed.]

QB: Mod. 19 Nov U.F. $x = x(t)$ satisfies $\boxed{2x^{(3)} + 5x^{(2)} - x = 0}$.

Then $\mathsf{Y} := \begin{bmatrix} x \\ x' \\ x'' \end{bmatrix}$ satisfies $\mathsf{Y}' = \mathsf{M}\cdot\mathsf{Y}$, where M is this 3×3 matrix of numbers:

$$\mathsf{M} = \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}.$$

QC: Mod. 26 Nov Gamma fnc: $\boldsymbol{\Gamma}(5)=$ ____ and $\boldsymbol{\Gamma}(\frac{5}{2})=$ ____

For all real $x > 1$, our $\boldsymbol{\Gamma}()$ function satisfies recurrence relation

$\boldsymbol{\Gamma}(x) =$ ____.

Last DfyQ quiz of semester...