DfyQ MAP2302 **Quizzes Q** Thursday 25Jan2018

Number Sets. An expression such as $k \in \mathbb{N}$ (read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$") means that k is a natural number; a ***natnum***.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \dots\}$.

$\mathbb{Z}$ = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive ***ratnums*** and $\mathbb{Q}_-$ for the negative ratnums.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

An "***interval of integers***" $[b\,..\,c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e}\,..\,2\boldsymbol{\pi}] = \{3, 4, 5, 6\} = [3\,..\,6] = (2\,..\,6]$. We allow b and c to be $\pm\infty$; so $(-\infty\,..\,-1]$ is $\mathbb{Z}_-$.

Floor function: $\lfloor \boldsymbol{\pi} \rfloor = 3$, $\lfloor -\boldsymbol{\pi} \rfloor = -4$. Ceiling fnc: $\lceil \boldsymbol{\pi} \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2\boldsymbol{i}| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand Side*' of an eqn or inequality. LhS: '*left-hand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', but pt: '*point*', as in "a fixed-pt of a map".

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** fnc, defined for $x>0$, is $\log(x) := \int_1^x \frac{\mathrm{d}v}{v}$. Its inverse-fnc is $\exp()$. For $x>0$, then, $\exp(\log(x)) = x = \mathrm{e}^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(\mathrm{e}^t)$. PolyExp: '*Polynomial-times-exponential*'. E.g, $F(t) := [3 + t^2]\cdot\mathrm{e}^{4t}$ is a polyExp.

Prefix nt- means '*non-trivial*'. E.g "a *nt*-soln to $f' = 5f$ is $f(t) := \mathrm{e}^{5t}$; a *trivial* soln is $f \equiv 0$."

Phrases. WLOG: '*Without loss of generality*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. Use iff: '*if and only if*'.

IST: '*It Suffices to*' as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Factorial. Def: $n! := n\cdot[n-1]\cdot[n-2]\cdots2\cdot1$; so $0! = 1$.

Rising Fctrl: $[\![x \uparrow K]\!] := x\cdot[x+1]\cdot[x+2]\cdots[x+[K-1]]$,

Falling Fctrl: $[\![x \downarrow K]\!] := x\cdot[x-1]\cdot[x-2]\cdots[x-[K-1]]$,

for natnum K and $x \in \mathbb{C}$. E.g, $[\![K\downarrow K]\!] = K! = [\![1\uparrow K]\!]$.

N.B: For $n \in \mathbb{Z}$: If $K > n$ then $[\![n \downarrow K]\!] = 0$.

Note $[\![x \uparrow K]\!] = [\![x + [K-1] \downarrow K]\!]$.

DfyQ quizzes so far...

Q1: Mon. 22Jan U.F $y = y(t)$ satisfies $y'' + 3y' + 2y = 0$, with $y(0) = 2$ and $y'(0) = 11$. So $y(t) = A\mathrm{e}^{\alpha t} + B\mathrm{e}^{\beta t}$ where

$\alpha=$ ________, $\beta=$ ________, $A=$ ________, $B=$ ________.

Q2: Fri. 26Jan Fnc $y_\alpha(t) :=$ ____________________________

is the general soln to $\frac{\mathrm{d}y}{\mathrm{d}t} = 4y^2 t$. [*Hint:* SoV.]

The fnc satisfying init.-cond. $y_\alpha(1) = 1/5$ has $\alpha =$ ________.

Q3: Mon. 29Jan Blanks$\in\mathbb{R}$. So $\frac{1}{2+3\boldsymbol{i}} =$ ________ $+\boldsymbol{i}\cdot$[________].

Thus $\mathrm{Im}\left(\frac{5-\boldsymbol{i}}{2+3\boldsymbol{i}}\right) =$ ____________.

By the way, $|5 - 3\boldsymbol{i}| =$ ____________.

Q4: Wed. 31Jan For $t>0$, fnc $y_\alpha(t) :=$ ____________________

is the gen.soln to $y' + \left[\frac{5}{t}\cdot y\right] = t$. [*Hint:* FOLDE.]

Changing topics, $\mathrm{Im}\left(\frac{3+2\boldsymbol{i}}{7-\boldsymbol{i}}\right) =$ ______________.

Q5: Fri. 02Feb For $t>0$, fnc $y_\alpha(t) :=$ ______ is the gen.soln to $y' + \left[\frac{8}{t}\cdot y\right] = t^9$. [*Hint:* FOLDE.]

Changing topics: Number $\left[\boldsymbol{i}+\sqrt{3}\right]^{70} = x + \boldsymbol{i}y$, for real numbers $x=$ ______ and $y=$ ______.

Q6: Mon. 05Feb *A* particular soln $p=p(t)$ to

$$p' + 2p = 6t^2 + 8t + 1$$

is $p(t) =$ ______ $\cdot t^2 +$ ______ $\cdot t +$ ______.

With $v := \exp(-2+5\boldsymbol{i})$, then $|v| =$ ______.

This $|v|$ lies in circle the correct interval

$[0,\frac{1}{2})$, $[\frac{1}{2},1)$, $[1,2)$, $[2,4)$, $[4,8)$, $[8,\infty)$.

Q7: Fri. 09Feb A particular soln to $y' + y = 2t\mathrm{e}^t$ is $y(t)=$ ______.

Q8: Mon. 12Feb **A** soln (use PolyExp) to $y'' + y = [4t-2]\mathrm{e}^t$ is $y(t)=$ ______.

Q9: Wed. 14Feb **A** soln (use PolyExp) to $y'' + y = 2t^2\mathrm{e}^{-t}$ is $y(t)=$ ______.

QA: Fri. 16Feb DE $\left[\mathcal{N}(x,y)\cdot\frac{\mathrm{d}y}{\mathrm{d}x}\right] + \mathcal{M}(x,y) = 0$ is *exact*, where

$$\mathcal{N}(x,y) := \left[x^2 - 7\right] \quad\text{and}\quad \mathcal{M}(x,y) := 2xy + 3\mathrm{e}^{3x}.$$

Its solns $y = y(x)$ satisfy $\mathbf{F}(x, y(x)) = \text{Const}$, where $\mathbf{F}(x,y) =$ ______.

QB: Mon. 19Feb For CCLDOp $\mathsf{L} := \mathbf{D}^3 - 3\mathbf{D} + 2\mathbf{I}$ and thrice diff'able fnc f, note $\mathsf{L}(f\cdot\mathrm{e}^{2t}) = \mathsf{V}(f)\cdot\mathrm{e}^{2t}$, where CCLDOp V is

$\mathsf{V} =$ ______ $\mathbf{D}^3 +$ ______ $\mathbf{D}^2 +$ ______ $\mathbf{D} +$ ______ $\mathbf{I}$.

[Put the correct number in *each* of the four blanks; zero, one, fractions, and negative numbers are allowed.]

QC: Wed. 21Feb Bacteria with birth-multiplier $\mathbf{B} :: \frac{1}{\mathsf{min}}$ are in a petri dish with carrying capacity $\mathbf{C} :: \mathsf{oz}$. The population, $p(t) :: \mathsf{oz}$, satisfies the Logistic DE.

The DE is ______.

For *Hysteria* bacteria, $\mathbf{B} = \frac{1/5}{\mathsf{min}}$. This petri dish has $\mathbf{C}$=16oz, with initial population $\mathbf{p}_0 = 2\mathsf{oz}$. The time when *Hysteria* has reached half the carrying capacity

is ______ min $\approx$ ______ min.

[NB: You may use exp() and log() to express your answer.]

QD: Fri. 02Mar With $f(x) := x^2$ and $g(x) := \sin(3x)$, then

$[f \circledast g](5) =$ ______. "For all cts h and φ:

$[h+\varphi]^{\circledast 2} = h^{\circledast 2} + 2[h \circledast \varphi] + \varphi^{\circledast 2}$." *T* *F*

QE: Mon. 12Mar With $G(x) := \sin(\sin(x))$, a soln to $y'' - y = G$ is $y := f \circledast G$, where $f(x) =$ ______.

For all integrable g and h, it is the case that $g \circledast h = h \circledast g$: *T* *F*

QF: Wed. 14Mar With $G(x) := \sin(\sin(x))$, a soln to $5y''' = G$ is $y := f \circledast G$, where $f(x) =$ ______.

Matrix-product $\begin{bmatrix}2 & -3\\ 4 & 5\end{bmatrix}\cdot\begin{bmatrix}7 & 0\\ 2 & 3\end{bmatrix} =$ ______.

QG: Fri. 16Mar Let $\mathsf{S} := \begin{bmatrix}x+1 & 2x\\ 1-x^2 & 3\end{bmatrix}$ and $\mathsf{B} := \begin{bmatrix}5 & 2 & 3\\ -1 & 3 & 0\\ 2 & 1 & 1\end{bmatrix}$. Then

Det(S)= ______ & Det(B)= ______.

QH: Fri. 23Mar

Acting on $y=y(t)$, DiffOp $\mathsf{E}(y) := t^2y'' - ty' + y$ is linear. Fnc $Y(t) := t$ satisfies $\mathsf{E}(Y) = 0$. Then Roo gives us a $Z(t) =$ .... satisfying $\mathsf{E}(Z) = 0$ *and* Z is L.I of Y.

Roo also produces a function $\varphi(t) =$ .... s.t $\mathsf{E}(\varphi) = t^{1/2}$.

QI: Wed. 18Apr Let $\mathsf{J} := \begin{bmatrix} 2 & 1 & \\ & 2 & 1 \\ & & 2 \end{bmatrix}$ and $\mathsf{C} := \begin{bmatrix} 3 & 1 & \\ 2 & 1 & \\ & & 1 \end{bmatrix}$. Note $\mathsf{C}^{-1} = \begin{bmatrix} 1 & -1 & \\ -2 & 3 & \\ & & 1 \end{bmatrix}$.

So

$\mathsf{e}^{\mathsf{J}t} =$ .... and $\mathsf{M} := \mathsf{CJC}^{-1} =$ ....

Finally, $\left[(2,3)\text{-entry of } \mathsf{e}^{\mathsf{M}t}\right] =$ ....

End of Quizzes Q