Number Sets. An expression such as $k \in \mathbb{N}$ (read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$") means that k is a natural number; a ***natnum***.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \dots\}$.

$\mathbb{Z}$ = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive ***ratnums*** and $\mathbb{Q}_-$ for the negative ratnums.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

An "***interval of integers***" $[b \,..\, c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e} \,..\, 2\boldsymbol{\pi}] = \{3, 4, 5, 6\} = [3 \,..\, 6] = (2 \,..\, 6]$. We allow b and c to be $\pm\infty$; so $(-\infty \,..\, -1]$ is $\mathbb{Z}_-$.

Floor function: $\lfloor \boldsymbol{\pi} \rfloor = 3$, $\lfloor -\boldsymbol{\pi} \rfloor = -4$. Ceiling fnc: $\lceil \boldsymbol{\pi} \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2\boldsymbol{i}| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand Side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', but pt: '*point*', as in "a fixed-pt of a map".

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** fnc, defined for $x>0$, is $\log(x) := \int_1^x \frac{\mathrm{d}v}{v}$. Its inverse-fnc is $\exp()$. For $x>0$, then, $\exp(\log(x)) = x = \mathrm{e}^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(\mathrm{e}^t)$. PolyExp: '*Polynomial-times-exponential*'. E.g, $F(t) := [3 + t^2]\cdot \mathrm{e}^{4t}$ is a polyExp.

Prefix nt- means '*non-trivial*'. E.g "a *nt*-soln to $f' = 5f$ is $f(t) := \mathrm{e}^{5t}$; a *trivial* soln is $f \equiv 0$."

Phrases. WLOG: '*Without loss of generality*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. Use iff: '*if and only if*'.

IST: '*It Suffices to*' as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Factorial. Def: $n! := n\cdot[n-1]\cdot[n-2]\cdots 2\cdot 1$; so $0! = 1$.

Rising Fctrl: $[\![x \uparrow K]\!] := x\cdot[x+1]\cdot[x+2]\cdots[x+[K-1]]$,

Falling Fctrl: $[\![x \downarrow K]\!] := x\cdot[x-1]\cdot[x-2]\cdots[x-[K-1]]$,

for natnum K and $x \in \mathbb{C}$. E.g, $[\![K \downarrow K]\!] = K! = [\![1 \uparrow K]\!]$.

N.B: For $n \in \mathbb{Z}$: If $K > n$ then $[\![n \downarrow K]\!] = 0$.

Note $[\![x \uparrow K]\!] = [\![x + [K-1] \downarrow K]\!]$.

DfyQ quizzes so far...

Q1: Wed. 27Sep *A* particular soln $y = y(t)$ to

$$\dagger: \quad [\mathbf{D} - 5\mathbf{I}]^3(y) = e^{5t} + e^{3t}$$

is $y(t) =$ ______.

Blanks∈ℝ. So $\frac{1}{2+3\boldsymbol{i}}$= ______ $+\boldsymbol{i}\cdot$[______].

Q2: Fri. 29Sep *A* particular soln $y=y(t)$ to

$$\dagger: \quad y'' - 6y' + 9y = e^{3t} + 5 \cdot e^{7t}$$

is $y(t) =$ ______.

Q3: Mon. 02Oct *A* particular soln $y=y(t)$ to

$$\dagger: \quad y'' - 8y' + 16y = t^5 e^{4t} + 2e^t$$

is $y(t) =$ ______.

Q4: Wed. 04Oct Blanks∈ℝ. So $\frac{1}{3-4\boldsymbol{i}}$= ______ $+\boldsymbol{i}\cdot$[______].

Thus $\dfrac{1-\boldsymbol{i}}{3-4\boldsymbol{i}}$ = ______ $+\boldsymbol{i}\cdot$[______].

By the way, $|5 - 3\boldsymbol{i}|$ = ______.

Q5: Mon. 09Oct *Am I in class today?*

circle one *"Yes!"* *"Of course!"*

Q6: Fri. 13Oct A tank initially has 80gal of salinity $2\frac{\text{lb}}{\text{gal}}$ brine. Pipe-1 feeds the tank, at rate $3\frac{\text{gal}}{\text{min}}$, with salinity $1\frac{\text{lb}}{\text{gal}}$ brine. Pipe-2 feeds at $2\frac{\text{gal}}{\text{min}}$ with salinity $2\frac{\text{lb}}{\text{gal}}$. The tank discharges brine at $9\frac{\text{gal}}{\text{min}}$. Until the tank empties, it holds

$W(t)$=[______]gal; it empties in ______ min.

The amount, $y(t)$, of lb of salt in the tank at time t, satisfies FOLDE $\frac{dy}{dt} + C(t) \cdot y = G(t)$, where $C(t)$= ______ and $G(t)$= ______.

QBonus: Mon. 23Oct Operators $\mathsf{V}, \mathbf{P}, \mathbf{Q}, \mathbf{R}, \mathbf{S}$ map from $\mathbf{C}^\infty \to \mathbf{C}^\infty$, and V is linear. The other maps are

$$\mathbf{P}(f) := [t \mapsto f(t) + 3], \quad \mathbf{Q}(f) := [t \mapsto f(t+3)],$$
$$\mathbf{R}(f) := [t \mapsto f(f(t))], \quad \mathbf{S}(f) := \mathsf{V}(\mathsf{V}(f)),$$

Then... **P** *is linear:* *T F*. **Q** *is linear:* *T F*. **R** *is linear:* *T F*. **S** *is linear:* *T F*.

$[x^3 \circledast x^2]$= ______. $[x^K \circledast x^N]$= ______

Q7: Wed. 25Oct In Lake Alice, the population, $p(t)$:: lb, of algae follows the logistic model. The carrying-capacity of Alice is 50 lb, and the initial algae population is 12 lb.

The birth-rate-mult, **B**, has units ______.

ITOf 12 lb, **B**, and 50 lb, the *DE* that $p()$ satisfies is ______.

Q8: Mon. 6Nov Determinant of $\mathsf{M} := \begin{bmatrix} 3 & 4 \\ 7 & 5 \end{bmatrix}$ is ______.

Op $\mathsf{L}(y) := t^2 y'' + 5ty' + 3y$ is equidimensional. A fnc $y \neq 0$ satisfying $\mathsf{L}(y) = 0$, is $y(t) =$ ______.

Q9: Wed. 08Nov By defn, $[f \circledast g](2017)$= ______.

Op $\mathsf{L}(y) := 3t^2 y'' + 5ty' - y$ is equidim'nal. The gen.soln to $\mathsf{L}(y)$=0 is $y(t) = \alpha \cdot$ ______ $+ \beta \cdot$ ______.

QA: Fri. 17Nov Please compute Wronskian

$\mathcal{W}(t, t+5, e^{2t})$= ______

QB: Mon. 27Nov From the integral defn,

$$\Gamma(\sqrt{7}) := \int_{\_\_\_}^{\_\_\_} \text{\_\_\_\_\_\_} \, dt$$

As a product (no integrals): $\mathbf{\Gamma}(\frac{9}{2}) =$ ________________________________.

QC: Wed. 29Nov Let $\mathsf{A} := \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$, $\mathsf{M} := \begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$ and $\mathsf{R} := \mathsf{MAM}^{-1}$. Then

the $(2, 2)$-entry of $\mathsf{e}^{\mathsf{R}t}$ is ________________________________.

QD: ?? Fri. 01Dec Ameliorate some of the World's Problems.