DfyQ
MAP2302 4689

Quizzes Q

30Aug2013

Abbrevs. WLOG: '*Without loss of generality*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. Use iff: '*if and only if*'.

IST: '*It Suffices to*' as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Q1: Fri. 30 Aug Function $h()$ satisfies $h'' - 4h' - 5h = 0$, and initial conditions $\boxed{h(0) = 2}$ and $\boxed{h'(0) = 3}$. So

$$h(t) = \alpha e^{At} + \beta e^{Bt}, \quad \text{for numbers}$$

$\alpha =$ ______, $A =$ ______, $\beta =$ ______, $B =$ ______.

Q2: Wed. 04 Sep For $x > 0$, let $B(x) := x^{\sin(3x)}$. Hence its derivative is $B'(x) = B(x) \cdot M(x)$, where $M(x)$ equals

______.

[*Hint:* How is y^z, for $y > 0$, defined ITOf the exponential fnc?]

Q3: Wed. 11 Sep Function y satisfies FOLDE

$$y' + [\tfrac{1}{x} \cdot y] = 6x\,.$$

Its gen.soln is $y(x) =$ ______.

The specific solution satisfying $\boxed{y(1) = 14}$ is

$y(x) =$ ______.

Q4: Wed. 18 Sep With $f(t) := \int_{\sin(5t)}^{7} \log(\cos(x))\, dx$, then $f'(t)$ equals

______.

Simplified, $f'(0) =$ ______.

[*Hint:* Chain rule and Fund. Thm of Calculus.]

Q5: Fri. 20Sep DE $[\mathcal{N}(x, y) \cdot \frac{dy}{dx}] + \mathcal{M}(x, y) = 0$ is *exact*, where

$$\mathcal{N}(x, y) := [x^2 - 7] \quad \text{and} \quad \mathcal{M}(x, y) := 2xy + 3e^{3x}\,.$$

Its soln $y = y(x)$ satisfies $\mathbf{F}(x, y(x)) = \text{Const}$, where $\mathbf{F}(x, y) =$ ______.

Q6: Mon. 23Sep DE $[2xy \cdot \frac{dy}{dx}] + [2 + 3x]y^2 = 0$ is not, alas, *exact*. Happily, multiplying both sides by (non-constant) fnc $W(x) =$ ______

gives a *new* DE which is exact. **Did you Check?**

Q7: Mon. 30Sep DE $[[2xy + 8y] \cdot \frac{dy}{dx}] + 4y^2 = 0$ is not, alas, *exact*. Happily, multiplying both sides by (non-constant) fnc $W(x) =$ ______

gives a *new* DE which is exact. **Did you Check?**

Q8: Wed. 02Oct Blanks$\in \mathbb{R}$. So $\frac{1}{2+3\boldsymbol{i}} =$ ______ $+\boldsymbol{i} \cdot [$ ______ $]$.

Thus $\dfrac{5 - \boldsymbol{i}}{2 + 3\boldsymbol{i}} =$ ______ $+\boldsymbol{i} \cdot [$ ______ $]$.

By the way, $|5 - 3\boldsymbol{i}| =$ ______.

Q9: Fri. 04Oct Binomial coefficient $\binom{8}{5, 3} =$ ______ $=$ ______.

Also,

$\dfrac{7\boldsymbol{i}}{4 - 3\boldsymbol{i}} =$ ______ $+\boldsymbol{i} \cdot [$ ______ $]$.

QA: Mon. 14Oct DE $h'' - 2h' + 10h = 0$, has fund.-set of solns $\{e^{\alpha t}, e^{\beta t}\}$, for complex numbers $\alpha =$ ______ and $\beta =$ ______.

Alternatively, we can write our fund.-set as

$$e^{Jt} \cdot \cos(Kt) \quad \text{and} \quad e^{Jt} \cdot \sin(Kt)\,,$$

for *real* numbers $J =$ ______ and $K =$ ______.

QB: Wed. 16Oct Number $6 \cdot \exp\left(\boldsymbol{i} \cdot \frac{5\boldsymbol{\pi}}{3}\right)$ equals $x + y\boldsymbol{i}$ for reals

$x=$ .... and $y=$ ....

With $v := \exp(-2+5\boldsymbol{i})$, then $|v| =$ ....

And $|v|$ lies in [circle the correct interval]

$[0, \frac{1}{2}),\quad [\frac{1}{2}, 1),\quad [1,2),\quad [2,4),\quad [4,8),\quad [8,\infty)$.

QC: Mon. 06Nov We can re-write function

$$f(t) \quad := \quad 2\cdot\cos\left(\tfrac{11}{6}\boldsymbol{\pi} + 4t\right) + \sqrt{3}\cdot\cos\left(\boldsymbol{\pi} + 4t\right)$$

as $f(t) = R\cdot\cos(\theta + 4t)$, for **real** numbers $R=$ .... $\geqslant 0$ and $\theta=$ .... $\in [0, 2\boldsymbol{\pi})$.

[*Hint:* OYOP, write cos() as the real-part of exp(*something*), and *Draw Yourself a large Useful Picture* in the complex plane.]

Soln. Let $\alpha = \frac{11}{6}\boldsymbol{\pi}$ and $T := \sqrt{3}$. Note $f(0)$ equals the real part of

$$V \;:=\; 2\mathrm{e}^{\alpha\boldsymbol{i}} + T\mathrm{e}^{\boldsymbol{\pi}\boldsymbol{i}} \overset{\text{note}}{=\!=} 2\mathrm{e}^{\alpha\boldsymbol{i}} - T \overset{\text{note}}{=\!=} -\boldsymbol{i}\,.$$

Thus $R = |-\boldsymbol{i}| = 1$ and $\theta = \operatorname{Arg}(-\boldsymbol{i}) = \frac{3}{2}\pi$.

QD: Wed. 20Nov Matrix $\mathsf{G} := \begin{bmatrix} 6 & -3 & 1 \\ 12 & -6 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is nilpotent. Computing, $\mathsf{G}^2=$ ....

The $(1,3)$-entry of $\mathrm{e}^{\mathsf{G}t}$ is ....

The $(2,2)$-entry of $\mathrm{e}^{\mathsf{G}t}$ is ....

QE: Mon. 02Dec The Laplace transform of fnc $f(t) := \cos(7t)$ is

$\widehat{f}(s) =$ ....

For IVP $3y'' - y = \cos(7t)$ with $y(0)=2$ and $y'(0)=5$, then,

$\widehat{y}(s) =$ ....

That's All, Folks!