Combo
MAD4203 3214 **Quizzes Q** 01Oct2012

Abbrevs. WLOG: '*Without loss of generality*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. Use iff: '*if and only if*'.

IST: '*It Suffices to*' as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Q1: Wed. 12 Sep Our invariant property for LBolt is that each row n satisfies: ______.

LBolt gives $G := \mathrm{Gcd}(221, 187) =$ ______. And $221S + 187T = G$, where $S=$ ______ & $T=$ ______ are integers.

Q2: Fri. 14 Sep Mod K:=534, the recipr. $\langle\frac{1}{37}\rangle_K =$ ______ $\in [0..K)$. So $x=$ ______ $\in [0..K)$ solves $3 - 37x \equiv_K 7$.

Q3: Wed. 19 Sep Mod N:=43, the recipr. $\langle\frac{1}{30}\rangle_N =$ ______ $\in [0..N)$. So $y=$ ______ $\in [0..N)$ solves $1 - 30y \equiv_N 8$.

Binomial-coeff $\binom{8}{5} =$ ______. (Single numeral)

Q4: Fri. 21 Sep The number of ways of having 4 objects from 8 types is $\left[\!\left[\begin{matrix}4\\8\end{matrix}\right]\!\right] \underset{\text{coeff}}{\overset{\text{Binom}}{=\!=}} \binom{\quad}{\quad}$ ______ $\underset{\text{numeral}}{\overset{\text{Integer}}{=\!=}}$ ______.

And $[\![{4 \atop 8}]\!] = [\![{N \atop T}]\!]$, where $N=$ ______ $\neq 4$, and $T=$ ______.

Q5: Mon. 24 Sep A polynomial $f{:}\mathbb{Z}_3 \to \mathbb{Z}_3$ of form $f(x) := \sum_{k=0}^{3} C_k x^k$ satisfies $[\forall z \in \mathbb{Z}_3{:}\ f(z) \equiv z^2]$, yet $C_1 \not\equiv 0$. So

$C_0 \equiv$ ______, $C_1 \equiv$ ______, $C_2 \equiv$ ______, $C_3 \equiv$ ______.

The physics lab has atomic *zinc, tin, silver* and *gold*. I'm allowed to take 6 atoms, so I have [expressed as single integer] ______ many possibilities.

Q6: Mon. 01 Oct The number of permutations of "PREPPER" is ______.

Prof. King believes that writing in complete, coherent sentences is crucial in communicating Mathematics, improves posture, and whitens teeth. Circle one:

True! **Yes!** ***wH'at S a?sEnTENcE***

Q7: Fri. 5 Oct There are $\binom{K}{J}$ many [diagonal] lattice-paths from point $(0, 2)$ to $(21, 7)$, where $K=$ ______ and $J=$ ______.

Such a path is ***bad***, if it touches the x-axis. And $|\mathrm{BAD}| = \binom{N}{L}$, where $N=$ ______ and $L=$ ______.

Q8: Mon. 26 Nov Perm $\alpha \in \mathbb{S}_7$ is $\circlearrowright 7\,4\,2\,5\,6\,3\,1 \circlearrowleft$. A particular permutation β satisfying $\beta^3 = \alpha$, is

$\beta = \circlearrowright 7$ ______ $\circlearrowleft$.

And $\mathrm{Sgn}(\beta)$ is (circle): **+1** **−1**.

Q9: Fri. 30 Nov Let $\mathbb{D}_N$ be the set of derangements in $\mathbb{S}_N$. Then $|\mathbb{D}_4| =$ ______.

The *set* of ***good*** k, st. $\mathbb{D}_k$ has both odd perms and even perms, is ______

Quizzes for Spring 2013 start here: Immediately below are quizzes given in Combo2. Below that are potential problems. (Not all quiz problems come from the potential problems.)

Q10: Mon. 28 Jan Writing $1/[1+2x]^5$ as $\sum_{n=0}^{\infty} B_n \cdot x^n$, then $B_{100} = M \cdot \binom{U}{D}$ where $M =$ ______, $U =$ ______, $D =$ ______.

Q11: Wed. 30 Jan Writing $1/[1-x^4]^9$ as $\sum_{n=0}^{\infty} B_n \cdot x^n$, then $B_{100} = \binom{U}{D}$ where $U =$ ______ and $D =$ ______.

Q12: Wed. 13 Feb a Coeff of x^{15} in $1/\left[1+2x^5\right]^3$ is ______.

b Bipartite graph $K_{3,2}$ is Eulerian. T F

Q13: Wed. 13 Feb a Graph G, a C_4 with a diagonal, has chromatic polynomial
$\mathcal{P}_G(x) =$ ______.

b Our G has ______ acyclic orientations.

Q14: Fri. 22 Mar Let D be K_5 but with an edge (but no vertices) deleted. Then $\mathcal{P}_D(x)$ equals [written in chromatic-factored form]
______.

Our D has ______ acyclic orientations.

Potential problems

GF1: Writing $1/[1-x^3]^8$ as $\sum_{n=0}^{\infty} B_n \cdot x^n$, then $B_{42} = \binom{U}{D}$ where $U =$ ______ and $D =$ ______.

GF2: Suppose $G(x)$ is the OGF of seq. $\vec{\mathbf{b}} = (b_0, b_1, \dots)$, where b_n is the number of partitions of n whose parts are primes <6. Then $G(x) =$ ______.

N1: Recall that a "***partition*** of a positive integer N" is an ordered tuple $(a_1, a_2, \dots, a_k)$ of *positive integers* such that $k \in \mathbb{Z}_+$ and $a_1 \geqslant a_2 \geqslant \dots \geqslant a_k$, and

$$a_1 + a_2 + \dots + a_k \;=\; N\,.$$

(Numbers $a_1, \dots, a_k$ are called the ***parts*** of the partition.) The partitions of N=3 are: 3, $2+1$, $1+1+1$.

i List the partitions of N=5: ______
______

ii OYOP: *In grammatical English sentences, write your essay on every third line* (usually)*, so that I can easily write between the lines.*

Recall that the *conjugate* of a partition is the partition you get by exchanging rows and columns in its Ferrers diagram. So the conjugate of partition $[5+3]$ is $[2+2+2+1+1]$.

A partition is ***self-conjugate*** if it equals its conjugate. Prove the following theorem from class (and from our text):

> THEOREM. *For each positive integer N: Let D_N be the number of partitions of N into distinct odd parts. Let S_N be the number of self-conjugate partitions of N. Then $D_N = S_N$.*

N2: By Binomial Series thm, the coeff of x^6 in $\sqrt[5]{1+x^2}$ is $\frac{n}{d}$, where integers $n \perp d$. So $n=$ ______ and $d=$ ______. (Write each naturally as a product of integers).

N3a: The Bell-number recurrence relation we discussed in class is

$$\forall K \in \mathbb{N}:\; B(K+1) \;=\; \sum_{n=\ell}^{K} [\mu_n \cdot B(n)]\,, \text{ where}$$

$\ell=$ ______ and $\mu_n=$ ______.
[N.B: The μ_n numbers may depend on K.]

N3b: *Define* the "n^{th} ***Bell number***". *State* the recurrence relation among the Bell numbers. Carefully *prove* this relation.

SNfk1: For natnums N, K, define $\mathbf{c}(N,K)$, the "***signless Stirling number*** of the first kind". Prove: THM: *For posints $N \geqslant K$, recurrence*

$$\mathbf{c}(N,K) \;=\; [N-1]\cdot\mathbf{c}(N-1,K) \;+\; \mathbf{c}(N-1,K-1)$$

holds.

SNfk2: Prove: THM: *For each posint N, polynomial $\sum_{k=0}^{N} \mathbf{c}(N,k)\cdot x^k$ equals $x\cdot[x+1]\cdot[x+2]\cdot\ldots\cdot[x+N-1]$.*

TTT: The GoF, "Game of fifteen", has two players. They alternate turns removing a number from the set $\{1, 2, \dots, 9\}$ and putting it in their own pile. When a player has, in his pile, three distinct numbers that sum to 15, he wins.

Prove that this game is isomorphic to tic-tac-toe.

Alg1: The Klein-4 group has ______ many group-automorphisms.

§A Potential quiz problems

Some of these may appear on quizzes/exams; naturally, with different data. (We haven't covered some of this material yet; earlier problems are potentially earlier.) *Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.*

LB1: Mod $K{:=}51$, the recipr. $\langle\frac{1}{20}\rangle_K =$ ______ $\in[0\,..\,K)$.

[*Hint:* ⨍] So $x=$ ______ $\in[0\,..\,K)$ solves $5-20x \equiv_K 2$.

LB2: Mod $K{:=}4203$, the recipr. $\langle\frac{1}{3214}\rangle_K =$ ______ $\in[0\,..\,K)$.

[*Hint:* ⨍] So $y=$ ______ $\in[0\,..\,K)$ solves $4-3214y \equiv_K 7$.

BC1: Binomial coefficient $\binom{7}{4} =$ ______ $=$ ______.

Multinomial coefficient $\binom{9}{4,2,3} =$ ______ $=$ ______.

[*Note:* Write your ans. ITOf factorials, then **also** write it as a single integer, or product of two, **without** factorials.]

BC2: Coeff of x^5y^{18} in $[x+1+3y]^{30}$ is ______.

[You may leave your answer as a product of *posints*, or you may multiply-out.]

Equality $\binom{74}{26,25,23} = \binom{74}{25}\cdot\binom{N}{K}$ suggests that $N=$ ______ and $K=$ ______.

Card: Given sets with cardinalities $|B|=7$ and $|E|=5$, the number of non-constant fncs in B^E is ______.

PS1: $\mathcal{P}(\mathcal{P}(\#\text{suits in a deck}))$ has ______ many elements.

LR1: Sequence $\vec{\mathbf{L}} := (L_n)_{n=0}^{\infty}$ is defined by $L_0 := 0$, $L_1 := 1$, and $\boxed{\forall n\in\mathbb{N}:\ L_{n+2} = 3L_{n+1} + L_n}$. This implies $\boxed{\forall k\in\mathbb{N}:\ L_k = [P\cdot\alpha^k + Q\cdot\beta^k]}$, for real numbers $\alpha=$ ______ $>\beta=$ ______, $P=$ ______, $Q=$ ______.

Rel1: As a single numeral, ______ is t.fol sum:

$$1-3\cdot\binom{9}{1}+9\cdot\binom{9}{2}-27\cdot\binom{9}{3}+81\cdot\binom{9}{4}\mp\ldots-3^9\cdot\binom{9}{9}.$$

Rel2: On $\mathbb{Z}_+$, write $x\,\$\,y$ IFF $xy<0$. So $\$$ is *Circle*

Transitive: *T F*. **Symm.:** *T F*. **Reflex.:** *T F*.

On $\mathbb{Z}$, say that $x \nabla y$ IFF $x-y \leqslant 1$. Then ∇ is:

Trans.: *T F*. **Symm.:** *T F*. **Reflex.:** *T F*.

(Be *careful* on <u>both</u> parts!)

Rel3: On $\Omega := [1\,..\,29]\times[1\,..\,29]$, define binary-relation $\mathbf{C}$ by: $(x,\alpha)\,\mathbf{C}\,(y,\beta)$ IFF $x\cdot\beta \equiv_{30} y\cdot\alpha$. Statement "*Relation* $\mathbf{C}$ *is an* ***equivalence relation***" is: *T F*

Both $\sim$ and $\bowtie$ are equiv-relations on a set Ω. Define binrels $\mathbf{I}$ and $\mathbf{U}$ on Ω as follows.

Define $\omega\,\mathbf{U}\,\lambda$ IFF *<u>Either</u>* $\omega\sim\lambda$ *<u>or</u>* $\omega\bowtie\lambda$ [or both].

Define $\omega\,\mathbf{I}\,\lambda$ IFF *<u>Both</u>* $\omega\sim\lambda$ *<u>and</u>* $\omega\bowtie\lambda$.

So "$\mathbf{U}$ *is an equiv-relation*" is: *T F*

So "$\mathbf{I}$ *is an equiv-relation*" is: *T F*

EF1: Euler $\varphi(121000) =$ ______.

Express your answer as a product $p_1^{e_1}\cdot p_2^{e_2}\cdot\ldots$ of *<u>primes</u>* to posint powers, with $p_1 < p_2 < \ldots$.

LS1: a Suppose $y\in\mathrm{QR}_N$, where N is oddprime. You compute BÃ©zout mults U and V st. $yU+NV=1$. Then "*U is a mod-N square*" is: *AT AF Nei*

b With $p := 323$, and $H := \frac{p-1}{2}$, note $66^H \equiv_p$ -2. Thus p is ______.