Number Sets. Expression $k \in \mathbb{N}$ [read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$"] means that k is a natural number; a ***natnum***. Expression $\mathbb{N} \ni k$ [read as "$\mathbb{N}$ ***owns*** k"] is a synonym for $k \in \mathbb{N}$.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \dots\}$.

$\mathbb{Z}$ = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive rationals and $\mathbb{Q}_-$ for the negative rationals.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

For $\omega \in \mathbb{C}$, let "$\omega > 5$" mean "ω ***is real*** and $\omega > 5$". [Use the same convention for $\geqslant, <, \leqslant$, and also if 5 is replaced by any real number.]

An "***interval of integers***" $[b \,..\, c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e} \,..\, 2\pi] = \{3, 4, 5, 6\} = [3 \,..\, 6] = (2 \,..\, 6]$. We allow b and c to be $\pm\infty$; so $(-\infty \,..\, -1]$ is $\mathbb{Z}_-$. And $[-\infty \,..\, -1]$, is $\{-\infty\} \cup \mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2\mathbf{i}| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'. RoC: '*Radius of Convergence*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand Side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', but pt: '*point*', as in "a fixed-pt of a map".

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** fnc, defined for $x>0$, is $\log(x) := \int_1^x \frac{dv}{v}$. Its inverse-fnc is $\exp()$. For $x>0$, then, $\exp(\log(x)) = x = \mathrm{e}^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(\mathrm{e}^t)$.

PolyExp: '*Polynomial-times-exponential*'; e.g, $[3 + t^2]\cdot\mathrm{e}^{4t}$. PolyExp-sum: '*Sum of polyexps*'. E.g, $f(t) := 3t\mathrm{e}^{2t} + [t^2]\cdot\mathrm{e}^t$ is a polyexp-sum.

Phrases. WLOG: '*Without loss of generality*'. IFF: '*if and only if*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. And ※="Contradiction".

IST: '*It Suffices to*' as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. inter alia: '*among other things*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Bi/Multi-nomial coeffs. For a natnum n, use "$n!$" to mean "*n factorial*"; the product of all posints $\leqslant n$. So $3! = 3\cdot 2\cdot 1 = 6$ and $5! = 120$. Also $0! = 1 = 1!$.

For natnum B and arb. complex number α, define

Rising Fctrl: $[\![\alpha\uparrow \mathsf{B}]\!] := \alpha\cdot[\alpha+1]\cdot[\alpha+2]\cdots[\alpha+[\mathsf{B}-1]]$,
Falling Fctrl: $[\![\alpha\downarrow \mathsf{B}]\!] := \alpha\cdot[\alpha-1]\cdot[\alpha-2]\cdots[\alpha-[\mathsf{B}-1]]$.

E.g, $[\![\mathsf{B}\downarrow\mathsf{B}]\!] = \mathsf{B}! = [\![1\uparrow\mathsf{B}]\!]$. Two further examples,

$$\left[\!\!\left[\tfrac{2}{7}\downarrow 4\right]\!\!\right] = \frac{2}{7}\cdot\frac{-5}{7}\cdot\frac{-12}{7}\cdot\frac{-19}{7} \quad\textit{and}\quad [\![1\downarrow 3]\!] = 1\cdot 0\cdot -1 = 0\,.$$

In particular, for $n\in\mathbb{N}$: If $\mathsf{B}>n$ then $[\![n\downarrow\mathsf{B}]\!]=0$. We pronouce $[\![5\downarrow\mathsf{B}]\!]$ as "5 falling-factorial B".

Binomial. The ***binomial coefficient*** $\binom{7}{3}$, read "7 choose 3", means *the number of ways of choosing 3 objects from 7 distinguishable objects*. Emphasising putting 3 objects in our left pocket and the remaining 4 in our right, we may write the coeff as $\binom{7}{3,4}$. [Read as "7 choose 3-comma-4."] Evidently

$$\dagger:\quad \binom{\mathsf{N}}{j} \overset{\text{with } k:=\mathsf{N}-j}{=\!=\!=} \binom{\mathsf{N}}{j,\ k} = \frac{\mathsf{N}!}{j!\cdot k!} = \frac{[\![\mathsf{N}\downarrow j]\!]}{j!}.$$

Note $\binom{7}{0} = \binom{7}{0,7} = 1$. Finally, the Binomial theorem says

$$£:\qquad [x+y]^{\mathsf{N}} = \sum_{j+k=\mathsf{N}} \binom{\mathsf{N}}{j,k}\cdot x^j y^k\,,$$

where (j,k) ranges over all *ordered* pairs of natural numbers with sum N.

For natnum N, binomials satisfy this addition law:

$$*:\quad \binom{\mathsf{N}+1}{\mathsf{B}+1} = \overbrace{\binom{\mathsf{N}}{\mathsf{B}}}^{\text{Pick last object.}} + \overbrace{\binom{\mathsf{N}}{\mathsf{B}+1}}^{\text{Avoid last object.}}\,.$$

Extending this to *all* $\mathsf{B}\in\mathbb{Z}$ forces:

$$\binom{\mathsf{N}}{\mathsf{B}} = 0, \qquad \textit{when } \mathsf{B}>\mathsf{N} \textit{ or } \mathsf{B} \textit{ negative}.$$

Case B>N is automatic in formula $\binom{\mathsf{N}}{\mathsf{B}} = \frac{[\![\mathsf{N}\downarrow\mathsf{B}]\!]}{\mathsf{B}!}$.

Multinomial. In general, for natnums $\mathsf{N} = k_1+\ldots+k_P$, the ***multinomial coefficient*** $\binom{\mathsf{N}}{k_1,\,k_2,\,\ldots,\,k_P}$ is the number of ways of partitioning N objects, by putting k_1 objects in pocket-one, k_2 objects in pocket-two, …putting k_P objects in the P^{th} pocket. Easily

$$‡:\quad \binom{\mathsf{N}}{k_1,k_2,\ldots,k_P} = \frac{\mathsf{N}!}{k_1!\cdot k_2!\cdot\ldots\cdot k_P!}\,.$$

And $[x_1+\cdots+x_P]^{\mathsf{N}}$ indeed equals the sum of terms

$$££:\qquad \binom{\mathsf{N}}{k_1,\ldots,k_P}\cdot {x_1}^{k_1}\cdot {x_2}^{k_2}\cdots {x_P}^{k_P}\,,$$

taken over all natnum-tuples $\vec{k}=(k_1,\ldots,k_P)$ that sum to N.

Define the sum $S_\ell := k_1+k_2+\ldots+k_\ell$. Then multinomial LhS(‡) equals this product of binomials:

$$\binom{\mathsf{N}}{k_1}\cdot\binom{\mathsf{N}-S_1}{k_2}\cdot\binom{\mathsf{N}-S_2}{k_3}\cdot\ldots\cdot\binom{\mathsf{N}-S_{P-1}}{k_P}.$$

[The last term is $\binom{k_P}{k_P} \overset{\text{note}}{=\!=} 1$.]

Operations on Sets. Use $\in$ for "is an element of". E.g, letting $\mathbb{P}$ be the set of primes, then, $5\in\mathbb{P}$ yet $6\notin\mathbb{P}$. Changing the emphasis, $\mathbb{P}\ni 5$ ["$\mathbb{P}$ *owns* 5"] yet $\mathbb{P}\not\ni 6$ ["$\mathbb{P}$ *does-not-own* 6"]

For subsets A and B of the same space, Ω, the ***inclusion relation*** $A\subset B$ means:

$$\forall\omega\in A,\ \textit{necessarily}\ B\ni\omega.$$

And this can be written $B\supset A$. Use $A\subsetneqq B$ for *proper* inclusion, i.e, $A\subset B$ yet $A\neq B$.

The *difference set* $B\smallsetminus A$ is $\{\omega\in B \mid \omega\notin A\}$. Employ A^c for the ***complement*** $\Omega\smallsetminus A$. Use $A\triangle B$ for ***symmetric difference*** $[A\smallsetminus B]\cup[B\smallsetminus A]$. Furthermore

$A \mathbin{\blacksquare} B$,	Sets A & B have *at least* <u>one</u> point in common; they intersect.
$A \sqcap B$,	The sets have *no* common point; disjoint.

The symbol "$A\mathbin{\blacksquare}B$" both asserts intersection <u>and</u> represents the set $A\cap B$. For a collection $\mathcal{C}=\{E_j\}_j$ of sets in Ω, let the ***disjoint union*** $\bigsqcup_j E_j$ or $\bigsqcup(\mathcal{C})$ represent the union $\bigcup_j E_j$ *and also* asserts that the sets are pairwise disjoint.

See next page…

Algebra [2019t] quizzes so far...

P1: Mon. 09 Sep May lightning ↯ strike this table! (I.e, please fill in.)

n	r_n	q_n	s_n	t_n
0	174	—	**1**	**0**
1	51		**0**	**1**
2				
3				
4				
5				

So GCD(174, 51)= ____ =[________ · 174] + [________ · 51].

P2: Mod. 16 Sep Dihedral group $\mathbb{D}_{12}$, the symmetries of a dodecagon, is generated by a rotation R and a F. Product

$$\mathtt{F}\,\mathtt{R}^2\,\mathtt{F}^2\,\mathtt{R}^{-3}\,\mathtt{F}\,\mathtt{R}^{-5}\,\mathtt{F}^3\,\mathtt{R}^7 \;=\; \mathtt{R}^J\,\mathtt{F}^K,$$

where $J =$ ______ $\in [0\,..\,12)$ and $K =$ ____ $\in \{0,1\}$.

P3: Fri. 27 Sep Euler $\varphi(121000) =$ ______________________.

Express your answer as a product $p_1{}^{e_1} \cdot p_2{}^{e_2} \cdot \ldots$ of *primes* to posint powers, with $p_1 < p_2 < \ldots$.

P4: Wed. 02 Oct Perm $\gamma \in \mathbb{S}_{10}$ is $(4\,3\,2\,1\,0)(9\,8\,7\,6\,5)$.
The possible cycle-types of β, where $\beta^2 = \gamma$, are ________________. And our γ has ________________ many sqroots.

P5: Fri. 11 Oct *Am I in class today?*

circle one *"Yes!"* *"Of course!"*

P6: Wed. 16 Oct $^{\#}\mathrm{Inn}(\mathbb{D}_{10}) =$ ________ and $^{\#}\mathrm{Aut}(\mathbb{D}_{10}) =$ ________.

P7: Mod. 04 Nov In $\mathbb{S}_4$, define $\alpha := (1,2,3)$ and $\beta := (3,4)$. Compute commutator $[\![\alpha, \beta]\!] := \alpha\beta\alpha^{-1}\beta^{-1} =$ ________________.

P8: Fri. 22 Nov The OP-isometry group, G, of the tetrahedron, **T**, has $|G| =$ ________. K-coloring the edges of **T**, there are

____ $\cdot [K^6 +$ ________________ $]$ G-distinct colorings.
[Burnside's thm, P.474.]

P9: !! Mon. 25 Nov Solve some of the World's Problems.

P: Wed. 27 Nov *No class!*