Q1: Frid. 5Feb $\int_0^{\pi/4} \sin(x)^3 \cdot \cos(x)\, dx =$ ________.

QB: Tues. 9Feb The quotient and remainder polynomials,

$q(x) =$ ________

& $r(x) =$ ________,

satisfy $B = [q \cdot C] + r$ and $\operatorname{Deg}(r) < \operatorname{Deg}(C)$, where $B(x) := x^4 + 5x^3 + 7x^2 + 3$ and $C(x) := x + 2$.

Q2: Frid. 12Feb $\int_3^{\infty} 1/[x^{17}]\, dx =$ ________.

Q3: Wedn. 17Feb By l'Hôpital's thm or other means, please compute

$\lim_{x \to 0} \frac{e^x - 1}{3x - 1} =$ ________. $\lim_{x \to \frac{\pi}{2}} \frac{\sin(2x)}{x - \frac{\pi}{2}} =$ ________.

Q4: Tues. 30Mar Compute the sum of this geometric series: $\sum_{n=4}^{\infty} [-1]^n \cdot -2/3^n =$ ________.

Q5: Mond. 12Apr For natural number K, the sum $\sum_{n=6}^{19+K} \left[\frac{-1}{4}\right]^n$ equals ________.

Q6: Frid. 16Apr Blanks$\in \mathbb{R}$. So $\frac{1}{3-4i} =$ ________ $+ i \cdot$ [________].

And $\frac{5+i}{3-4i} =$ ________ $+ i \cdot$ [________].

By the way, $|5 - 3i| =$ ________.

Q7: Tues. 20Apr $\sum_{n=0}^{\infty}$ ________ $= 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots =$ ________.

Set $w := [3 + i]^2$. So $|w| =$ ________.

$\operatorname{Im}\left(\frac{1}{w}\right) =$ ________. $\operatorname{Im}(e^w) =$ ________.

The note said: Remainder term in a Taylor series? N^{th}-coeff in a Taylor series? Complex exponential or sin or cos fnc? Argand plane, geometric interpretation of complex-arith? What is e^{4+3i}, real and imaginary parts? Solve problems C2, C3a, C3b, GS9, GS10.

§A Potential quiz problems

Some of these may appear on quizzes; naturally, with different data. *Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.*

pf1 $\frac{A}{x-1} + \frac{Bx+C}{x^2+1} = \frac{6x^2 - 7x + 3}{[x-1][x^2+1]}$ with $B =$ ____, $C =$ ____.

pf2 $\frac{Ex+F}{x^2+x+1} + \frac{Gx+H}{[x^2+x+1]^2} = \frac{x^3 + 3x^2 + 6x + 6}{[x^2+x+1]^2}$. with numbers $G =$ ________, $H =$ ________.

pf3 $\frac{A}{x-4} + \frac{B}{x+3} + \frac{C}{x+3} = \frac{2x^2 - 7x + 2}{[x-4][x+3][x+3]}$ with $C =$ ____ $\in \mathbb{Q}$.

Int1 $\int_2^{\infty} \frac{z}{3^z}\, dz =$ ________.

Int2 $\int \sqrt{5t^4 + t^2}\, dt =$ ________.

Int3 $\int_5^{\infty} \frac{1}{x \cdot [\log(x)]^3}\, dx =$ ________

and $\int_5^{\infty} \frac{1}{x \cdot \log(x)}\, dx =$ ________. [Subst.]

Arc1 Compute the **arclength** of the curve $P(t) := \left(3^t \cos(t), 3^t \sin(t)\right)$ from $t = \pi$ to $t = 2\pi$. Arclength= ________.

Arc2 Compute the **arclength** of the curve $\mathbf{w}(t) := t^2 \hat{\imath} + [\cos(t) + t\sin(t)]\hat{\jmath} + [\sin(t) - t\cos(t)]\hat{\mathbf{k}}$ from $t = 0$ to $t = \pi$. Arclength= ________.

Arc3 The graph of $y = \frac{1}{12x} + x^3$, for $x \in (1, 2)$, has length= ________.

Int4 $\int_2^7 \sqrt{1 + x^2}\, dt =$ ________. [*Hint:* Be careful!]

That a spiral is ***equi-angular*** precisely means that ________ [Imagine 4 blank lines].

The ***tautochrone*** curve satisfies (picture and explanation) [Imagine 5 blank lines]. ________

The ***brachistochrone*** curve satisfies (picture and explanation) [Imagine 5 blank lines]. ________

A polynomial $h(x)$ is ***monic*** IFF ________ [Imagine 3 blank lines]. ________

A polynomial $g(x, y, z)$ is ***homogeneous*** IFF ____ [Imagine 5 blank lines]. ________

CoM1 The width of the (parallel) top and bottom edges of a trapezoid **T** are W and $W+12$, and its height is 18. So Area(**T**)= ________.
Geometrically, $\lim_{W \nearrow \infty} Y_W =$ ________ and $Y_0 =$ ________, where $Y_W \stackrel{\text{note}}{=}$ ________ denotes the distance of Centroid(**T**) above Bottom(**T**).

CoM2 Glued to a massless plate is a 10 lb weight at the origin, a 15 lb weight at the point $(3, -1)$, and 5 lb at point (________ , ________), thus putting the center-of-mass of the weighted-plate at $(2, 1)$.

CoM3 In a disk, of radius 5ft, you drill a radius 2ft hole whose edge passes through the center of the disk. How far from the center of the *hole* is the centroid of the holed-disk? ________.

Some new quiz/exam problems:

GS0 $\sum_{n=0}^{2} r^n = \frac{19}{25}$. So $r=$ ________ or **DNE**.

GS1 Compute the sum of this geometric series: $\sum_{\beta=3}^{\infty} [-1]^\beta \cdot [3/5]^\beta =$ ________.

GS2 For natural number K, the sum $\sum_{n=3}^{3+K} 4^n$ equals ________.

GS3 $\sum_{k=1}^{\infty} r^k = \frac{5}{8}$. So $r=$ ________ or **DNE**.
[*Hint:* The sum starts with k at **one**, not zero.]

GS4 Express this sum as a rational in lowest terms. $\sum_{n=1}^{\infty} \frac{2^n - 5^n}{10^n} =$ ________.

GS5a $\sum_{L=0}^{\infty} r^L = \frac{4}{9}$. So $r=$ ________ or **DNE**.

GS5b $\sum_{J=0}^{\infty} \omega^J = \frac{7}{9}$. So $\omega=$ ________ or **DNE**.

GS6 $\sum_{n=2}^{\infty} r^n = 1$. So $r=$ ________ or **DNE**.
[*Hint:* The sum starts with n at **two**, not zero.]

GS7 $\sum_{n=-1}^{\infty} y^n = \frac{-4}{3}$. So $y=$ ________ or **DNE**.
[*Hint:* The sum starts with n at **negative-one**, not zero.]

GS8 Compute the sum of this geometric series: $\sum_{n=2}^{\infty} [1/7]^{3n} =$ ________.

TS1 Sum $\sum_{k=2}^{\infty} \frac{1}{k[k+3]} = \frac{U}{D}$, where $U=$ ______ and $D=$ ______ are co-prime posints.

[*Hint:* Telescoping series (several). "Co-prime" means "with no common factor".]

TS2 Let $b_n := \exp(\frac{4}{n+5}) \overset{\text{note}}{=\!=} e^{4/[n+5]}$. Then $\sum_{n=4}^{\infty} [b_n - b_{n+1}] =$ ______.

[*Hint:* A real, $+\infty$, $-\infty$ or **DNEin**$\overline{\mathbb{R}}$.]

TS3 Let $b_n := \frac{\log_2(n) + n}{1 + 2n}$. Then $\sum_{n=2}^{\infty} [b_n - b_{n+1}] =$ ______.

[*Hint:* A real, $+\infty$, $-\infty$ or **DNEin**$\overline{\mathbb{R}}$.]

TS4 Let $b_n := [-1]^n \cdot \cos\left(\frac{[-1]^n}{n}\right)$. Then $\sum_{n=2}^{\infty} [b_n - b_{n+1}] =$ ______.

[*Hint:* A real, $+\infty$, $-\infty$ or **DNEin**$\overline{\mathbb{R}}$.]

TS5 Let $b_n := \log\left(\frac{5+n}{3+n^2}\right)$. Then $\sum_{n=-2}^{\infty} [b_n - b_{n+1}] =$ ______.

[*Hint:* A real, $+\infty$, $-\infty$ or **DNEin**$\overline{\mathbb{R}}$.]

Al1 Series $\sum_{n=2}^{\infty} \frac{[-1]^n}{n \cdot \log(n)}$ (*circle one*): **diverges, converges-conditionally, converges-absolutely**.

[*Hint:* Altenating-series-test and Integral-test.]

Al2 Let $g(x) := x^2 - x - 1$ and $b_n := 1/n^4$. Then $\sum_{n=4}^{\infty} [-1]^n \cdot g(b_n) =$ ______.

[A real, $+\infty$, $-\infty$ or **DNEin**$\overline{\mathbb{R}}$.] [*Hint:* Roots of g are? And $\lim(\vec{\mathbf{b}})$ is?]

Al3 Let $f(x) := x^2 - x - 1$ and $b_n := 5^{\frac{n+1}{2n}}$. Then $\sum_{n=4}^{\infty} [-1]^n \cdot f(b_n) =$ ______.

[A real, $+\infty$, $-\infty$ or **DNEin**$\overline{\mathbb{R}}$.] [*Hint:* Roots of f are? And $\lim(\vec{\mathbf{b}})$ is?]

C1 Sum $\sum_{n=4}^{\infty} \frac{\sin(n) + \sqrt{n}}{\cos(n) + n}$ converges in $\mathbb{R}$. Circle *T* *F*

[*Hint:* Limit-comparison test.]

C2 Sum $\sum_{n=1}^{\infty} \frac{5 + \sin(n)}{[3 - \cos(n)]n^2}$ converges in $\mathbb{R}$. Circle *T* *F*

[*Hint:* Comparison test.]

Create a series. Evidently, answers are not unique for the following questions. As always, **DNE** may be the correct answer.

Cr1 Sum $\sum_{j=1}^{\infty} a_j = \infty$ yet $\sum_{j=1}^{\infty} [-1]^j \cdot a_j < \infty$, where posreal $a_j :=$ ______.

Cr2 Sum $\sum_{j=1}^{\infty} b_j = \infty$ yet $\sum_{j=1}^{\infty} b_j^2 < \infty$, where posreal $b_j :=$ ______.

Cr3 Sum $\sum_{j=1}^{\infty} c_j^2 = \infty$ yet $\sum_{j=1}^{\infty} c_j < \infty$, where posreal $c_j :=$ ______.

Cr4 Sum $\sum_{j=1}^{\infty} \theta_j = \infty$ yet $\sum_{j=1}^{\infty} \sin(\theta_j) < \infty$, where posreal $\theta_j :=$ ______.

Cr5 Sum $\sum_{j=1}^{\infty} \sin(\varphi_j) = \infty$ yet $\sum_{j=1}^{\infty} \varphi_j < \infty$, where posreal $\varphi_j :=$ ______.

Polar coordinates

PC1 Set $\mathbf{A} := \frac{4\pi}{3}$. Let C_1 be the polar-curve $r = \frac{1}{\cos(\theta)}$, and C_2 the polar-curve $r = 1/\cos(\mathbf{A} + \theta)$. The *unique* intersection point of these curves is (r_0, θ_0), where $\theta_0 =$ ______ and $0 < r_0 =$ ______.

PC2 For scale-factor $S>0$, let $\mathcal{N}_S$ be the number of **distinct** intersection-points of polar-curve $r = \frac{S}{\cos(\theta)}$ with polar-curve $r = 2\cos(\theta)$. So $\mathcal{N}_3=$ ______,
$\mathcal{N}_2=$ ______, $\mathcal{N}_1=$ ______ and $\mathcal{N}_{-1} =$ ______.

PC3 Inside polar-curve $\text{RadialDist}(\theta) = \theta^2$, the area from ray $\theta = 0$ to $\theta = \frac{\pi}{2}$ is ______.

[*Hint:* See P.650 in Stewart's text.]

PC4 The area inside *one lobe* of polar-curve $r = 4\cos(7\theta)$, is ______.

[*Hint:* Recall $[\cos(\alpha)]^2 = \frac{1}{2}[1 + \cos(2\alpha)]$.]

PC5 Inside polar-curve $\text{RadialDist}(\theta) = \frac{1}{\cos(\theta)}$, the area from ray $\theta = \frac{\pi}{6}$ to $\theta = \frac{\pi}{2}$ is ______.

[*Hint:* Graph the "curve"; the area is evident w/o integration]

PC6 The area *inside* of polar-curve $r = 8\cos(\theta)$, yet *outside* of $r = 4$, is ______.

[*Hint:* Graphing the curves makes this easy.]

PC7 *One lobe* of polar-curve $r = 3\sin(5\theta)$, has area

$$\int_0^{\ldots\ldots} \left[\ldots\ldots\ldots\ldots \right] \cdot d\theta.$$

Powers series

Henceforth let Mac abbreviate *Maclaurin series*.

PS1 Power series $\sum_{n=3}^{\infty} \frac{[3x]^n}{n+7}$ has RoC= ______.

PS2 The Mac of $\frac{5}{1-x^3}$ has RoC= ______.
Mac(x)= ______.
Write the first 5 non-zero terms,
e.g, $8x^3 + \frac{1}{8}x^6 + \frac{3}{2}x^8 - x^{12} - 7x^{15} + \cdots$.

PS3 The Mac of $\frac{t}{t^2+7}$ has RoC= ______.
Mac(t)= ______.
Write the first 5 non-zero terms,
e.g, $8t^3 + \frac{1}{8}t^6 + \frac{3}{2}t^8 - t^{12} - 7t^{15} + \cdots$.

PS4 The Mac of $\log(1-3x)$ has RoC= ______.
Mac(x)= ______.
Write the first 5 non-zero terms,
e.g, $8x^3 + \frac{1}{8}x^6 + \frac{3}{2}x^8 - x^{12} - 7x^{15} + \cdots$.

PS5 Power series $f(x) = \sum_{n=3}^{\infty} \frac{[3x-6]^n}{4+[2n]^3}$ has
RoC= ______. And BoC= ______.

PS6 The Mac of $x^3 e^x$ has RoC= ______.
Mac(x)= ______.
Write the first 5 non-zero terms,
e.g, $8x^3 + \frac{1}{8}x^6 + \frac{3}{2}x^8 - x^{12} - 7x^{15} + \cdots$.

PS7 Power series $g(t) = \sum_{n=3}^{\infty} \frac{n! \cdot [t+2]^n}{n^3 + 7}$ has
RoC= ______. And BoC= ______.

PS8 Power series $h(x) = \sum_{n=3}^{\infty} \frac{n! \cdot [x-2]^n}{n^n}$ has
RoC= ______. And BoC= ______.

C1 Blanks$\in\mathbb{R}$. So $\frac{1}{2+3\boldsymbol{i}} =$ ______ $+\boldsymbol{i}\cdot$[______].
And $\frac{7-2\boldsymbol{i}}{2+3\boldsymbol{i}} =$ ______ $+\boldsymbol{i}\cdot$[______].
By the way, $|5-3\boldsymbol{i}| =$ ______.

C2 In $\mathbb{R}$: $[1+\boldsymbol{i}]^{86} = [\ \ \ \ \ \ \ \ \ ] + \boldsymbol{i}\cdot[\ \ \ \ \ \ \ \ \ ]$.

[*Hint: Multiplying complexes multiplies their modulii, and adds their angles. You may use* sin *and* cos *if you wish.*]

C3a Write $\sqrt{3} + \boldsymbol{i}$ as $r\cdot\text{cis}(\theta)$, where $r=$ __________ $\in\mathbb{R}_+$ and $\theta=$ __________ $\in[0, \frac{\pi}{2}]$.

C3b Sqroot of $\boldsymbol{i}-1$ in the upper-halfplane is $r\cdot\text{cis}(\theta)$, where $r=$ __________ $\in\mathbb{R}_+$ and $\theta=$ __________ $\in[0, \frac{\pi}{2}]$.

GS9 Compute the sum of this geometric series: $\sum_{n=1}^{\infty}\left[\frac{4}{2+3\boldsymbol{i}}\right]^n =$ _________________________________.

GS10 Compute the sum of this geometric series: $\sum_{k=0}^{\infty}\left[\frac{2+3\boldsymbol{i}}{4}\right]^k =$ _________________________________.

C4 The std.pic. of $\mathbb{C}$ is called the __________ plane. Set $w := \log(3)\cdot[2+\boldsymbol{i}]$. So $|w|=$ ___________________. $\text{Re}(\mathsf{e}^w)=$ ______________________ and $|\mathsf{e}^w|=$ ________.