SeLo
MHF3202 2787

Quizzes S 30Aug2013

Number Sets. An expression such as $k \in \mathbb{N}$ (read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$") means that k is a natural number; a ***natnum***.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \ldots\}$.

$\mathbb{Z}$ = integers = $\{\ldots, -2, -1, 0, 1, \ldots\}$. For the set $\{1, 2, 3, \ldots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive ***ratnums*** and $\mathbb{Q}_-$ for the negative ratnums.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

An **"*interval of integers*"** $[b \,..\, c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e} \,..\, 2\pi] = \{3, 4, 5, 6\} = [3 \,..\, 6] = (2 \,..\, 6]$. We allow b and c to be $\pm\infty$; so $(-\infty \,..\, -1]$ is $\mathbb{Z}_-$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'.

Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand Side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', but pt: '*point*', as in "a fixed-pt of a map".

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'. CoV: '*Change-of-Variable*'.

Phrases. WLOG: '*Without loss of generality*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. Use iff: '*if and only if*'.

IST: '*It Suffices to*' as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Quizzes so far...

S1: Stmt $C \Rightarrow B$ has *contrapositive* ________ and *converse* ________. Recall &, $\vee$, $\neg$ mean And, Or, Not.

Using *only* symbols $\wedge, \vee, \neg, \boldsymbol{B}, \boldsymbol{C},], [$, write $C \Rightarrow B$ as ________.

S2: Write the free vars in each of these expressions.

$$\overbrace{\exists n \in \mathbb{N}\colon\; f(n) \subset \underbrace{\bigcup_{\ell=r-4}^{r+7} \underbrace{\{x \in \mathbb{Z} \mid \ell \cdot n \equiv_5 x^2\}}_{E1}}_{E2}}^{E3}$$

$E1$: ________. $E2$: ________. $E3$: ________.

S3: Fri. 28 Feb *What is the day-of-the-week, today?*

____

S4: Mon. 10 Mar LBolt gives $G := \mathrm{Gcd}(153, 27) =$ ________. And $153S + 27T = G$, where $S =$ ________ & $T =$ ________ are integers.

Potential future problems

Microquizzes *similar* to the following may appear at some point. Also, microquizzes *not* similar to these may appear...

I welcome and encourage you to post solutions to these problems to our SeLo Archive.

LB0: May lightning ↯ strike this table! (I.e, please fill in.)

n	r_n	q_n	s_n	t_n
0	100	—	**1**	**0**
1	23		**0**	**1**
2				
3				
4				
5				
6				

Thus $1 = [\ \_\_\_\_\_\_ \cdot 100] + [\ \_\_\_\_\_\_ \cdot 23]$.

LB1: LBolt gives $G := \text{Gcd}(133, 56) =$ ________. And $133S + 56T = G$, where $S=$ ________ & $T=$ ________ are integers.

LB2: LBolt: $\text{Gcd}(21, 15) =$ ______ $\cdot 21 +$ ______ $\cdot 15$.

So (LBolt again) $G := \text{Gcd}(21, 15, 35) =$ __________ and ______ $\cdot 21 +$ ______ $\cdot 15 +$ ______ $\cdot 35 = G$.

LB3: Our invariant property for LBolt is that each row n satisfies: ____________________________.

HS1: *Am I in class today?*

circle one *"Yes!"* *"Of course!"*

LB4: Mod $K:=50$, the recipr. $\langle \frac{1}{21} \rangle_K =$ ________ $\in [0..K)$.

[*Hint:* ↯] So $x=$ __________ $\in [0..K)$ solves $4 - 21x \equiv_K 1$.

MC1: The coeff of $x^3 y^2$ in $[x + 1 + 2y]^8$ is ____________________________.

[You may leave your answer as a product of *posints*, or you may multiply-out.]

R1: Suppose $\mathbf{S}$ and $\mathbf{T}$ are each transitive binary-relations on a set Ω. Then

Rel. $\mathbf{T} \circ \mathbf{S}$ *is transitive:* AT AF Nei

Rel. $\mathbf{S} \circ \mathbf{S}$ *is transitive:* AT AF Nei

Now suppose $\mathbf{A}$ is an antreflexive binrel on Ω. Then

Rel. $\mathbf{A} \circ \mathbf{A}$ *is anti-reflexive:* AT AF Nei

On $\Omega := [-9, 8]$, say $x \mathbf{R} y$ if $x \cdot y < 76$. So $\mathbf{R}$ is Circle *Reflexive* *Symmetric* *Transitive*.

R2: We consider binrels on $\Omega :=$ Stooges $:= \{M, L, C\}$. There are ______________ **Anti-reflexive** binrels, and ______________ **Reflexive** binrels, and ______________ **Symmetric** binrels. The number of **strict total-orders** is ______________.

∞1: Mon. 02Dec Between sets $\mathbf{S} := \mathbb{Z}_+$ and $\mathbf{\Omega} := \mathbb{N}$, consider injections $f : \mathbf{S} \hookrightarrow \mathbf{\Omega}$ and $h : \mathbf{\Omega} \hookrightarrow \mathbf{S}$, defined by

$$f(x) := 3x \quad \text{and} \quad h(y) := y + 5.$$

Schröder-Bernstein produces a set $G \subset h(\mathbf{\Omega}) \subset \mathbf{S}$ st., letting $U := \mathbf{S} \setminus G$, the fnc $\varphi : \mathbf{S} \twoheadrightarrow \mathbf{\Omega}$ is a *bijection*, where

$$*: \quad \varphi|_U := f|_U \quad \text{and} \quad \varphi|_G := h^{-1}|_G.$$

For this (f, h), the (G, U) pair is unique. Computing, $\varphi(17) =$ ______. $\varphi(137) =$ ______. $\varphi^{-1}(603) =$ ______

More to come...