SeLo
MHF3202 2787 **Quizzes S** 30Aug2013

Abbrevs.

Number Sets. An expression such as $k \in \mathbb{N}$ (read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$") means that k is a natural number; a ***natnum***.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \dots\}$.

$\mathbb{Z}$ = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive ***ratnums*** and $\mathbb{Q}_-$ for the negative ratnums.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

An "***interval of integers***" $[b\,..\,c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[e\,..\,2\pi] = \{3, 4, 5, 6\} = [3\,..\,6] = (2\,..\,6]$. We allow b and c to be $\pm\infty$; so $(-\infty\,..\,-1]$ is $\mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2i| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand Side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', _but_ pt: '*point*', as in "a fixed-pt of a map".

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** fnc, defined for $x>0$, is $\log(x) := \int_1^x \frac{dv}{v}$. Its inverse-fnc is $\exp()$. For $x>0$, then, $\exp(\log(x)) = x = e^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(e^t)$. PolyExp: '*Polynomial-times-exponential*'. E.g, $F(t) := [3 + t^2]\cdot e^{4t}$ is a polyExp.

Phrases. WLOG: '*Without loss of generality*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. Use iff: '*if and only if*'.

IST: '*It Suffices to*' as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. QED: *quod erat demonstrandum*, meaning "end of proof".

S1: Fri. 30 Aug Stmt $C \Rightarrow B$ has *contrapositive* ________ and *converse* ________. Recall &, $\vee$, $\neg$ mean AND, OR, NOT.

Using *only* symbols $\wedge, \vee, \neg, \boldsymbol{B}, \boldsymbol{C},], [$, write $C \Rightarrow B$ as ________.

S2: Mon. 30 Sep Write the free vars in each of these expressions.

$$\overbrace{\exists n \in \mathbb{N}:\ f(n) \subset \underbrace{\bigcup_{\ell=r-4}^{r+7} \underbrace{\{x \in \mathbb{Z} \mid \ell \cdot n \equiv_5 x^2\}}_{E1}}_{E2}}^{E3}$$

$E1$: ________. $E2$: ________. $E3$: ________.

S3: Wed. 18 Sep LBolt gives $G := \text{Gcd}(133, 56) =$ ________. And $133S + 56T = G$, where $S =$ ________ & $T =$ ________ are integers.

S4: Fri. 20Sep LBolt: $\text{Gcd}(21, 15) =$ ________ $\cdot 21 +$ ________ $\cdot 15$.

So (LBolt again) $G := \text{Gcd}(21, 15, 35) =$ ________ and ________ $\cdot 21 +$ ________ $\cdot 15 +$ ________ $\cdot 35 = G$.

S5: Mon 23Sep $\mathcal{B}$-number (Blip number) 22 is $\mathcal{B}$-irreducible: *T* *F*.

$\mathcal{B}$-numbers $K :=$ ________ and $N :=$ ________ are st. $22 \bullet\!| [K \cdot N]$, yet $22 \nmid K$ and $22 \nmid N$. Hence 22 is *not* $\mathcal{B}$-prime.

Also, $\mathcal{B}$-GCD(175, 70) = ________.

S6: Wed. 02Oct Mod K:=50, the recipr. $\langle \frac{1}{21} \rangle_K =$ ________ $\in [0\,..\,K)$.
[*Hint:* ↯] So $x =$ ________ $\in [0\,..\,K)$ solves $4 - 21x \equiv_K 1$.

S7: Mon. 07Oct Mod K:=153, the recipr. $\langle \frac{1}{10} \rangle_K =$ ________ $\in [0\,..\,K)$.

[*Hint:* ↯] So $x=$ ________ $\in[0..K)$ solves $7-10x \equiv_K 4$.

Coeff of x^2yz^5 in $\left[x+y+z\right]^8$ is ________.

[You may leave your answer as a product of *posints*, or you may multiply-out.]

S8: Wed. 09Oct Coeff of x^3y^2 in $\left[x+1+2y\right]^8$ is ________.

[You may leave your answer as a product of *posints*, or you may multiply-out.]

S9: Fri. 11Oct Euler $\varphi(121000) =$ ________.

Express your answer as a product ${p_1}^{e_1} \cdot {p_2}^{e_2} \cdot \ldots$ of *primes* to posint powers, with $p_1 < p_2 < \ldots$.

The last 2 digits of 37^{162} are: ____ ____.

SA: Wed. 16Oct Suppose $\mathbf{S}$ and $\mathbf{T}$ are each transitive binary-relations on a set Ω. Then

Rel. $\mathbf{T} \circ \mathbf{S}$ *is transitive:* AT AF Nei

Rel. $\mathbf{S} \circ \mathbf{S}$ *is transitive:* AT AF Nei

Now suppose $\mathbf{A}$ is an antreflexive binrel on Ω. Then

Rel. $\mathbf{A} \circ \mathbf{A}$ *is anti-reflexive:* AT AF Nei

On $\Omega := [-9, 8]$, say $x \mathbf{R} y$ if $x \cdot y < 76$. So $\mathbf{R}$ is Circle *Reflexive* *Symmetric* *Transitive*.

SB: Fri. 18Oct *Am I in class today?*

circle one *"Yes!"* *"Of course!"*

SC: Fri. 25Oct We consider binrels on Ω := Stooges := $\{M, L, C\}$.

There are ________ **Anti-reflexive** binrels, and ________ **Reflexive** binrels, and ________ **Symmetric** binrels. The number of **strict total-orders** is ________.

SD: Fri. 22Nov *Am I in class today?*

circle one *"Yes!"* *"Of course!"*

SE: Mon. 02Dec Between sets $\mathbf{S} \coloneqq \mathbb{Z}_+$ and $\mathbf{\Omega} \coloneqq \mathbb{N}$, consider injections $f\colon \mathbf{S} \hookrightarrow \mathbf{\Omega}$ and $h\colon \mathbf{\Omega} \hookrightarrow \mathbf{S}$, defined by

$$f(x) \coloneqq 3x \quad \text{and} \quad h(y) \coloneqq y + 5\,.$$

Schröder-Bernstein produces a set $G \subset h(\mathbf{\Omega}) \subset \mathbf{S}$ st., letting $U \coloneqq \mathbf{S} \smallsetminus G$, the fnc $\varphi\colon \mathbf{S} \hookrightarrow\!\!\!\!\rightarrow \mathbf{\Omega}$ is a *bijection*, where

$$*: \qquad \varphi\!\downharpoonright_U \coloneqq f\!\downharpoonright_U \quad \text{and} \quad \varphi\!\downharpoonright_G \coloneqq h^{-1}\!\downharpoonright_G\,.$$

For this (f, h), the (U, G) pair is unique. Computing,
$\varphi(17)=$ .... $\varphi(137)=$ .... $\varphi^{-1}(603)=$ ....

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That's all there is,
There ain't no more,
unless I meet
that bear once more.
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