

Abbrevs.

Number Sets. An expression such as $k \in \mathbb{N}$ (read as “ k is an element of \mathbb{N} ” or “ k in \mathbb{N} ”) means that k is a natural number; a *natnum*.

\mathbb{N} = natural numbers = $\{0, 1, 2, \dots\}$.

\mathbb{Z} = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the *posints*, use \mathbb{Z}_+ . Use \mathbb{Z}_- for the negative integers, the *negints*.

\mathbb{Q} = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use \mathbb{Q}_+ for the positive *ratnums* and \mathbb{Q}_- for the negative ratnums.

\mathbb{R} = reals. The *posreals* \mathbb{R}_+ and the *negreals* \mathbb{R}_- .

\mathbb{C} = complex numbers, also called the *complexes*.

An “*interval of integers*” $[b..c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[e..2\pi] = \{3, 4, 5, 6\} = [3..6] = (2..6]$. We allow b and c to be $\pm\infty$; so $(-\infty..-1]$ is \mathbb{Z}_- .

Mathematical objects. Seq: ‘sequence’. poly(s): ‘polynomial(s)’. irred: ‘irreducible’. Coeff: ‘coefficient’ and var(s): ‘variable(s)’ and parm(s): ‘parameter(s)’. Expr.: ‘expression’.

Fnc: ‘function’ (so ratfnc: means rational function, a ratio of polynomials). cty: ‘continuity’. cts: ‘continuous’. diff’able: ‘differentiable’.

Soln: ‘Solution’. Thm: ‘Theorem’. Prop’n: ‘Proposition’. CEX: ‘Counterexample’. eqn: ‘equation’. RhS: ‘RightHand Side’ of an eqn or inequality. LhS: ‘left-hand side’. Sqrt or Sroot: ‘square-root’, e.g, “the sroot of 16 is 4”. Ptn: ‘partition’, but pt: ‘point’, as in “a fixed-pt of a map”.

FTC: ‘Fund. Thm of Calculus’. IVT: ‘intermediate-Value Thm’. MVT: ‘Mean-Value Thm’. CoV: ‘Change-of-Variable’.

Phrases. WLOG: ‘Without loss of generality’. TFAE: ‘The following are equivalent’. ITOF: ‘In Terms Of’. OTForm: ‘of the form’. FTSOC: ‘For the sake of contradiction’. Use iff: ‘if and only if’.

IST: ‘It Suffices to’ as in ISTShow, ISTExhibit.

Use w.r.t: ‘with respect to’ and s.t: ‘such that’.

Latin: e.g: *exempli gratia*, ‘for example’. i.e: *id est*, ‘that is’. QED: *quod erat demonstrandum*, meaning “end of proof”.

S1: ^{Fri.}_{30 Aug} Stmt $C \Rightarrow B$ has *contrapositive* _____ and *converse* _____. Recall $\&, \vee, \neg$ mean AND, OR, NOT.

Using *only* symbols $\wedge, \vee, \neg, B, C,], [$, write $C \Rightarrow B$ as _____.

S2: ^{Mon.}_{30 Sep} Write the free vars in each of these expressions.

$$\exists n \in \mathbb{N}: f(n) \subset \underbrace{\bigcup_{\ell=r-4}^{r+7}}_{E2} \underbrace{\{x \in \mathbb{Z} \mid \ell \cdot n \equiv_5 x^2\}}_{E1}$$

E1: _____ . E2: _____ . E3: _____ .

S3: ^{Wed.}_{18 Sep} LBolt gives $G := \text{Gcd}(133, 56) =$ _____. And $133S + 56T = G$, where $S =$ _____ & $T =$ _____ are integers. _____

S4: ^{Fri.}_{20 Sep} LBolt: $\text{Gcd}(21, 15) =$ _____ $\cdot 21 +$ _____ $\cdot 15$. So (LBolt again) $G := \text{Gcd}(21, 15, 35) =$ _____ and _____ $\cdot 21 +$ _____ $\cdot 15 +$ _____ $\cdot 35 = G$. _____

S5: ^{Mon.}_{23 Sep} **S**-number (Stuart number) 22 is **S**-irreducible: $T F$. **S**-numbers $K :=$ _____ and $N :=$ _____ are st. $22 \nmid [K \cdot N]$, yet $22 \nmid K$ and $22 \nmid N$. Hence 22 is *not S*-prime.

Also, **S**-GCD(175, 70) = _____.

S6: ^{Wed.}_{02 Oct} Mod $K := 50$, the recipr. $\langle \frac{1}{21} \rangle_K =$ _____ $\in [0..K)$. [Hint: $\frac{1}{21}$] So $x =$ _____ $\in [0..K)$ solves $4 - 21x \equiv_K 1$. _____

S7: ^{Mon.}_{07 Oct} Mod $K := 153$, the recipr. $\langle \frac{1}{10} \rangle_K =$ _____ $\in [0..K)$. [Hint: $\frac{1}{10}$] So $x =$ _____ $\in [0..K)$ solves $7 - 10x \equiv_K 4$. _____

The coeff of $x^2 y z^5$ in $[x + y + z]^8$ is _____ [Hint: $\frac{1}{8}$] _____

S8: ^{Wed.}_{09Oct} The coeff of x^3y^2

in $[x + 1 + 2y]^8$ is

[You may leave your answer as a product of *posints*, or you may multiply-out.]

S9: ^{Fri.}_{11Oct} Euler $\varphi(121000) =$

Express your answer as a product $p_1^{e_1} \cdot p_2^{e_2} \cdot \dots$ of primes to posint powers, with $p_1 < p_2 < \dots$

The last 2 digits of 37^{162} are:

SA: ^{Wed.}_{16Oct} Suppose **S** and **T** are each transitive binary-relations on a set Ω . Then

Rel. **T** \circ **S** is transitive: *AT* *AF* *Nei*
 Rel. **S** \circ **S** is transitive: *AT* *AF* *Nei*

Now suppose **A** is an antireflexive binrel on Ω . Then
 Rel. **A** \circ **A** is anti-reflexive: *AT* *AF* *Nei*

On $\Omega := [-9, 8]$, say $x \mathbf{R} y$ if $x \cdot y < 76$. So **R**
 is Reflexive Symmetric Transitive.

SB: ^{Fri.}_{18Oct} *Am I in class today?*

"Yes!" "Of course!"

SC: ^{Fri.}_{25Oct} We consider binrels on $\Omega := \text{Stooges} := \{M, L, C\}$.

There are **Anti-reflexive** binrels,
 and **Reflexive** binrels,
 and **Symmetric** binrels. The
 number of **strict total-orders** is

SD: ^{Fri.}_{22Nov} *Am I in class today?*

"Yes!" "Of course!"

SE: ^{Mon.}_{02Dec} Between sets $\mathbf{S} := \mathbb{Z}_+$ and $\mathbf{\Omega} := \mathbb{N}$, consider injections $f: \mathbf{S} \hookrightarrow \mathbf{\Omega}$ and $h: \mathbf{\Omega} \hookrightarrow \mathbf{S}$, defined by

$$f(x) := 3x \quad \text{and} \quad h(y) := y + 5.$$

Schröder-Bernstein produces a set $G \subset h(\mathbf{\Omega}) \subset \mathbf{S}$ st., letting $U := \mathbf{S} \setminus G$, the fnc $\varphi: \mathbf{S} \hookrightarrow \mathbf{\Omega}$ is a *bijection*, where

$$*: \quad \varphi|_U := f|_U \quad \text{and} \quad \varphi|_G := h^{-1}|_G.$$

For this (f, h) , the (G, U) pair is unique. Computing,
 $\varphi(17) =$. $\varphi(137) =$. $\varphi^{-1}(603) =$.

That's all there is,
 There ain't no more,
 unless I meet
 that bear once more.