

Table of Laplace transforms

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General Lap xforms. For complex numbers α and β , let $\alpha \succ \beta$ mean $\text{Re}(\alpha) > \text{Re}(\beta)$

With μ an arbitrary real number and $f, g \in \text{Ord}(\mu)$,^{♥1} each formula in the table below holds for all complex $s \succ \mu$, unless a trailing note says otherwise.

Natural numbers: N, j, k .
Real numbers: F . Positive reals: 7, 9.
Complex Numbers: R, Q, B_j .

h or $h(t)$	\widehat{h} or $\widehat{h}(s)$
C: $f \otimes g$	$\widehat{f} \cdot \widehat{g}$
Mul: $f(9t)$	$\frac{1}{9} \cdot \widehat{f}(s/9)$
1: f'	$s \cdot \widehat{f}(s) - f(0)$
1_N: $f^{(N)}$	$s^N \widehat{f}(s) - \left[\sum_{j+k=N-1} s^j \cdot f^{(k)}(0) \right]$
1^{**}_{Ply}: $[q(\mathbf{D})](f)$	$q \cdot \widehat{f}$
1₋₁: $\int_0^t f(x) dx \stackrel{\text{note}}{=} [1 \otimes f](t)$	$\widehat{f}(s)/s$
2: $f(t) \cdot t$	$-[\widehat{f}]'$
2_N: $f(t) \cdot t^N$	$[-1]^N \widehat{f}^{(N)}$
2₋₁: $f(t)/t$	$\int_s^\infty \widehat{f}$
3: $f(t) \cdot e^{Rt}$	$\widehat{f}(s - R)$
4: $f(t-7) \cdot \mathbf{H}(t-7)$	$e^{-7s} \cdot \widehat{f}(s)$
$f(t) \cdot \mathbf{H}(t-7)$	$e^{-7s} \cdot \widehat{f(\widehat{t+7})}(s)$
5: f has period 7	$\mathcal{L}_7(f)(s) / [1 - e^{-7s}]$

The “**D**” in formula (1_{Ply}) is the differentiation operator. The “**q**” is a general degree- N polynomial

$$q(s) = B_0 + B_1s + B_2s^2 + \dots + B_Ns^N.$$

This^{**} needs that derivatives $f^{(k)}(0)$ are zero, for each $k \in [0..N)$. In that case, (1_{Ply}) is says that function

$$s \mapsto q(s) \cdot \widehat{f}(s)$$

^{♥1}See “exponential order” in the `laplace.xform.latex` file.

is the xform of $[B_0f + B_1f' + B_2f'' + \dots + B_Nf^{(N)}]$.
Line (5) arranges for periodic input fncs with

$$\mathcal{L}_7(f)(s) := \int_0^7 e^{-st} f(t) dt.$$

Formula (5) holds $\forall s \succ 0$.

Line (3) is valid for all $s \succ R + \mu$.

Formula (Mul) is valid for all $s \succ 9\mu$.

Specific Lap xforms. Many of the below come from the “General” table, with $f := \mathbf{1}$.

h or $h(t)$	\widehat{h} or $\widehat{h}(s)$
1	$1/s$
t^N	$N! / s^{N+1}$
12: t^R	$\Gamma(R+1) / s^{R+1}$
$1/\sqrt{t}$	$\sqrt{\pi} / \sqrt{s}$
\sqrt{t}	$\frac{1}{2} \sqrt{\pi} / s^{3/2}$
13: $e^{Rt} \cdot t^N$	$N! / [s - R]^{N+1}$
14: $e^{Rt} \cdot \sin(Ft)$	$\frac{F}{[s-R]^2 + F^2}$
$e^{Rt} \cdot \cos(Ft)$	$\frac{s-R}{[s-R]^2 + F^2}$

Formula (12) requires $R \succ -1$, as does this defn:

$$\Gamma(R+1) := \int_0^\infty t^R e^{-t} dt.$$

The (12) formulas hold for all $s \succ 0$.

Formulas (13,14) are valid for all $s \succ R$.

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