# Oft-used notation of Prof. JLF King

Jonathan L.F. King
*University of Florida, Gainesville FL 32611-2082, USA*
squash@ufl.edu
Webpage http://squash.1gainesville.com/

23 September, 2017 (at *16:20*)

## Names for sets of numbers

An expression such as $k \in \mathbb{N}$ (read as "$k$ is an element of $\mathbb{N}$" or "$k$ in $\mathbb{N}$") means that $k$ is a natural number; a ***natnum***.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \dots\}$.

$\mathbb{Z}$ = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive ***ratnums*** and $\mathbb{Q}_-$ for the negative ratnums.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

## Phrases used in proofs

WLOG: '*Without loss of generality*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. Use iff: '*if and only if*'.

IST: '*It Suffices to*' as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

**Latin:** e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. QED: *quod erat demonstrandum*, meaning "end of proof".

## Names for Mathematical Objects

Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'.

Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand Side*' of an eqn or inequality. LhS: '*left-hand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', <u>*but*</u> pt: '*point*', as in "a fixed-pt of a map".

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'. CoV: '*Change-of-Variable*'.

**Prefixes:** Use nv- for '*non-void*', e.g "the cartesian product of two nv-sets is non-void". Use nt- for '*non-trivial*', e.g "the (positive) nt-divisors of 14 are $2, 7, 14$, whereas the *proper* divisors are $1, 2, 7$".

## Operations on Sets

Use $\in$ for "is an element of". E.g, letting $\mathbb{P}$ be the set of primes, then, $5 \in \mathbb{P}$ yet $6 \notin \mathbb{P}$. Changing the emphasis, $\mathbb{P} \ni 5$ ("$\mathbb{P}$ *owns* 5") yet $\mathbb{P} \not\ni 6$.

For subsets $A$ and $B$ of the same space, $\Omega$, the ***inclusion relation*** $A \subset B$ means:

$$\forall \omega \in A, \text{ necessarily } B \ni \omega.$$

And this can be written $B \supset A$. Use $A \subsetneqq B$ for *proper* inclusion, i.e, $A \subset B$ yet $A \neq B$.

The *difference set* $B \setminus A$ is $\{\omega \in B \mid \omega \notin A\}$. Employ $A^c$ for the ***complement*** $\Omega \setminus A$. Use $A \triangle B$ for ***symmetric difference*** $[A \setminus B] \cup [B \setminus A]$. Furthermore

| | |
|---|---|
| $A \boxdot B$, | Sets $A$ & $B$ have *at least* <u>*one*</u> point in common; they intersect. |
| $A \sqcap B$, | The sets have *no* common point; disjoint. |

The symbol "$A \boxdot B$" both asserts intersection <u>and</u> represents the set $A \cap B$. For a collection $\mathcal{C} = \{E_j\}_j$ of sets in $\Omega$, let the ***disjoint union*** $\bigsqcup_j E_j$ or $\bigsqcup(\mathcal{C})$ represent the union $\bigcup_j E_j$ *and also* assert that the sets are pairwise disjoint.

If there is a *measure* on the space then

| | |
|---|---|
| $A \overset{\text{a.e}}{\sqcap} B$, | means their intersection is a nullset; it is empty a.e. (i.e *almost everywhere*) |

In contrast, $A \overset{\text{a.e}}{\boxdot} B$ means that the sets intersect in positive mass.

## Linear Algebra

In a real vectorspace $\mathbf{V}$, say that

$$\dagger: \quad \sum_{j=1}^{N} \alpha_j \mathbf{v}_j \quad (\text{with each } \alpha_j \in \mathbb{R})$$

is a ***linear combination*** (***lin.comb***) of vectors (points) $\mathbf{v}_1, \dots, \mathbf{v}_N$. If, further, these scalars satisfy

$$\ddagger: \quad \alpha_1 + \alpha_2 + \dots + \alpha_N = 1,$$

then we call (†) a ***weighted average*** of the points. Finally, if (‡) *and* each $\alpha_j \geqslant 0$, then we call (†) a ***convex average*** of the points.

Given a set $S \subset \mathbf{V}$ of points, we define three supersets

$$\operatorname{Spn}(S) \supset \operatorname{AffSpn}(S) \supset \operatorname{Hull}(S).$$

The ***span*** is the set of all lin.combs (†), as $\{\mathbf{v}_1, \ldots, \mathbf{v}_N\}$ ranges over *all finite* subsets of $S$. The ***affine span*** is the set of all (†) satisfying (‡), whereas the ***hull*** is the smaller set of all convex averages. Thus $\operatorname{Spn}(S)$ is the smallest *subspace* (that includes $S$) whereas $\operatorname{AffSpn}(S)$ is the smallest *affine-space* and $\operatorname{Hull}(S)$ is the smallest *convex set.*

A point $\mathbf{w} \in C$ is an "***extreme point*** of a convex set $C$" if: *Whenever we write* $\mathbf{w} = \alpha_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2$ *as a convex average* (of points $\mathbf{v}_1, \mathbf{v}_2 \in C$), *then necessarily* $\mathbf{v}_1 = \mathbf{v}_2 = \mathbf{w}$. A non-void set $C \subset \mathbf{V}$ is an $N$-dimensional ***simplex*** (an "$N$-simplex") if we can write it as

$$C = \operatorname{Hull}(\mathbf{w}_1, \ldots, \mathbf{w}_{N+1})$$

where no $\mathbf{w}_j$ is in the affine-span of the others. Equivalently, $C$ has precisely $N{+}1$ extreme-pts, and $\operatorname{Dim}(C) = N$.

## Polynomials

Use ***poly*** for "polynomial". An integer-coefficient poly is a $\mathbb{Z}$-poly or an ***intpoly***. With rational coeffs, it is a $\mathbb{Q}$-poly or ***ratpoly***. An $\mathbf{F}$-poly has its coeffs come from a *field* $\mathbf{F}$. (A commutative ring is ok too).

The poly ***zip*** has all of its coefficients zero. Say that a poly is 5-***topped*** if its degree is *strictly* less than 5. Over a field $\mathbf{F}$, the set of (single variable) $N$-topped polys forms an *$N$-dimensional vectorspace.*

## Counting

For a natnum $n$, use "$n!$" to mean "$n$ ***factorial***"; the product of all positive-integers less-equal $n$. So $3! = 3 \cdot 2 \cdot 1 = 6$ and $5! = 120$. Also $0! = 1$ and $1! = 1$.

The ***binomial coefficient*** $\binom{7}{3}$, read "7 choose 3", means *the number of ways of choosing* 3 *objects from* 7 *distinguishable objects.* If we think of putting these objects in our left pocket, and putting the remaining 4 things in our right pocket, then we write the coefficient as $\binom{7}{3,4}$. [Read as "7 choose 3-comma-4."] Note that $\binom{7}{0} = \binom{7}{0,7} = 1$. Also note this identity:

$$[x+y]^N = \sum_{j+k=N} \binom{N}{j,k} \cdot x^j y^k,$$

where $(j,k)$ ranges over all *ordered* pairs of natural numbers with sum $N$.

In general, for natnums $N = K_1 + \ldots + K_L$, the ***multinomial coefficient*** $\binom{N}{K_1, K_2, \ldots, K_L}$ means the number of ways of partitioning $N$ different things, by putting $K_1$ of them in pocket 1 and $K_2$ of them in pocket 2, and so on. Easily

$$\binom{N}{K_1, K_2, \ldots, K_L} = \frac{N!}{K_1! \cdot K_2! \cdots K_L!}.$$

And $[x_1 + \cdots + x_L]^N$ indeed equals the sum of

$$\binom{N}{K_1, \ldots, K_L} \cdot {x_1}^{K_1} \cdot {x_2}^{K_2} \cdots {x_L}^{K_L},$$

taken over all natnum-tuples $\vec{K} = (K_1, \ldots, K_L)$ that sum to $N$.

Let "50 ***placed into*** 3 ***types***" (or just "50 into 3 types") denote the number of ways of choosing 50 individual candies from 3 distinct *types* of candy. Write it $\left[\!\left[{50 \atop 3}\right]\!\right]$. For $N, T \in \mathbb{N}$, use $\left[\!\left[{N \atop T}\right]\!\right]$ for "$N$ (objects) into $T$-types", the number of ways of filling $T$ distinct types with $N$ objects total. So $\left[\!\left[{0 \atop T}\right]\!\right] = 1$ for each natnum $T$. And $\left[\!\left[{N \atop 0}\right]\!\right] = 0$ for each *posint* $N$. Furthermore

$$\begin{matrix}\forall T \in \mathbb{Z}_+ \\ \text{and } \forall N \in \mathbb{N}\end{matrix} : \quad \left[\!\!\left[{N \atop T}\right]\!\!\right] = \binom{N+T-1}{N,\ T-1} = \left[\!\!\left[{T-1 \atop N+1}\right]\!\!\right].$$

This, since $\binom{N+T-1}{N,\ T-1} = \binom{T-1+[N+1]-1}{T-1,\ [N+1]-1}$.

## Number Theory

Use $\equiv_N$ to mean "congruent mod $N$". Let $n \perp k$ mean that $n$ and $k$ are co-prime. Use $k \bullet\!| \; n$ for "$k$ divides $n$". Its negation $k \not\bullet\!| \; n$ means "$k$ does not divide $n$." Use $n \,|\!\bullet\, k$ and $n \,|\!\not\bullet\, k$ for "$n$ is/is-not a multiple of $k$." Finally, for $p$ a prime and $E$ a natnum: Use double-verticals, $p^E \bullet\!\| \; n$, to mean that $E$ is the ***highest*** power of $p$ which divides $n$. Or write $n \,\|\!\bullet\, p^E$ to emphasize that this is an assertion about $n$. Use ***PoT*** for Power of Two and ***PoP*** for Power of (a) Prime.

A natnum $N$ is a ***SOTS***, Sum Of Two Squares, if there are integers for which $\ell^2 + k^2 = N$. If there exists such a pair with $\ell \perp k$, then $N$ is ***coprime-SOTS***. (E.g, 25 has a non-coprime rep as $5^2+0^2$; nonetheless, 25 *is* coprime-SOTS, since $25 = 3^2 + 4^2$. OTOHand, both $4 = 0^2 + 2^2$ and $40 = 2^2 + 6^2$ have these unique SOTS reps, so neither is *coprime*-SOTS.) An odd integer $L$ is ***4Neg*** if $L \equiv_4 -1$ and is ***4Pos*** if $L \equiv_4 +1$. Fermat's Prime-SOTS Thm says: *Oddprime $p$ is SOTS iff $p$ is* 4Pos.

Mod $N$, a ***rono*** is a (square–)Root Of Negative One; an integer $I$ such that $I^2 \equiv_N -1$.

Use ***CRT*** for the Chinese Remainder Thm.

For $N$ a posint, use $\Phi(N)$ or $\Phi_N$ for the set $\{r \in [1 .. N] \mid r \perp N\}$. The cardinality $\boldsymbol{\varphi}(N) := |\Phi_N|$ is the ***Euler phi function***. (So $\boldsymbol{\varphi}(N)$ is the cardinality of the multiplicative group, $\Phi_N$, in the $\mathbb{Z}_N$ ring.) Use ***EFT*** for the Euler-Fermat Thm, which says: *Suppose that integers $b \perp N$, with $N$ positive. Then $b^{\boldsymbol{\varphi}(N)} \equiv_N 1$.*

Filename: `Texdir/LatexCode/nota.all.latex`
As of: *Tuesday 02Mar2004*. Typeset: *23Sep2017* at *16:20*.