

Oft-used notation of Prof. JLF King

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Names for sets of numbers

An expression such as $k \in \mathbb{N}$ (read as “ k is an element of \mathbb{N} ” or “ k in \mathbb{N} ”) means that k is a natural number; a *natnum*.

\mathbb{N} = natural numbers = $\{0, 1, 2, \dots\}$.

\mathbb{Z} = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the *posints*, use \mathbb{Z}_+ . Use \mathbb{Z}_- for the negative integers, the *negints*.

\mathbb{Q} = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use \mathbb{Q}_+ for the positive *ratnums* and \mathbb{Q}_- for the negative ratnums.

\mathbb{R} = reals. The *posreals* \mathbb{R}_+ and the *negreals* \mathbb{R}_- .

\mathbb{C} = complex numbers, also called the *complexes*.

Phrases used in proofs

WLOG: ‘Without loss of generality’. TFAE: ‘The following are equivalent’. ITOF: ‘In Terms Of’. OTForm: ‘of the form’. FTSOC: ‘For the sake of contradiction’. Use iff: ‘if and only if’.

IST: ‘It Suffices to’ as in ISTShow, ISTExhibit.

Use w.r.t: ‘with respect to’ and s.t: ‘such that’.

Latin: e.g. *exempli gratia*, ‘for example’. i.e. *id est*, ‘that is’. QED: *quod erat demonstrandum*, meaning “end of proof”.

Names for Mathematical Objects

Seq: ‘sequence’. poly(s): ‘polynomial(s)’. irred: ‘irreducible’. Coeff: ‘coefficient’ and var(s): ‘variable(s)’ and parm(s): ‘parameter(s)’. Expr.: ‘expression’. Col: ‘Constant of Integration’. Lol: ‘Limit(s) of Integration’.

Fnc: ‘function’ (so ratfnc: means rational function, a ratio of polynomials). cty: ‘continuity’. cts: ‘continuous’. diff’able: ‘differentiable’.

Soln: ‘Solution’. Thm: ‘Theorem’. Prop’n: ‘Proposition’. CEX: ‘Counterexample’. eqn: ‘equation’. RhS: ‘RightHand Side’ of an eqn or inequality. LhS: ‘left-hand side’. Sqrt or Sroot: ‘square-root’, e.g. “the sroot of 16 is 4”. Ptn: ‘partition’, *but* pt: ‘point’, as in “a fixed-pt of a map”.

FTC: ‘Fund. Thm of Calculus’. IVT: ‘intermediate-Value Thm’. MVT: ‘Mean-Value Thm’. CoV: ‘Change-of-Variable’.

Prefixes: Use nv- for ‘non-void’, e.g. “the cartesian product of two nv-sets is non-void”. Use nt- for ‘non-trivial’, e.g. “the (positive) nt-divisors of 14 are 2, 7, 14, whereas the *proper* divisors are 1, 2, 7”.

Operations on Sets

Use \in for “is an element of”. Letting \mathbb{P} be the set of primes, then, $5 \in \mathbb{P}$ yet $6 \notin \mathbb{P}$. Changing the emphasis, $\mathbb{P} \ni 5$ (“ \mathbb{P} owns 5”) yet $\mathbb{P} \not\ni 6$.

For subsets A and B of the same space, Ω , the *inclusion relation* $A \subset B$ means:

$$\forall \omega \in A, \text{ necessarily } \omega \in B.$$

And this can be written $B \supset A$. Use $A \subsetneq B$ for *proper* inclusion, i.e. $A \subset B$ yet $A \neq B$.

The *difference set* $B \setminus A$ is $\{\omega \in B \mid \omega \notin A\}$. Employ A^c for the *complement* $\Omega \setminus A$. Use $A \Delta B$ for *symmetric difference* $[A \setminus B] \cup [B \setminus A]$. Furthermore

$$\begin{array}{ll} A \cap B, & \text{Sets } A \text{ \& } B \text{ have at least one point in} \\ & \text{common; they intersect.} \\ A \cap B, & \text{The sets have no common point; dis-} \\ & \text{joint.} \end{array}$$

The symbol “ $A \cap B$ ” both asserts intersection and represents the set $A \cap B$. For a collection $\mathcal{C} = \{E_j\}_j$ of sets in Ω , let the *disjoint union* $\bigsqcup_j E_j$ or $\bigsqcup(\mathcal{C})$ represent the union $\bigcup_j E_j$ and also assert that the sets are pairwise disjoint.

If there is a *measure* on the space then

$$A \overset{\text{a.e.}}{\cap} B, \quad \text{means their intersection is a nullset; it is empty a.e. (i.e almost everywhere)}$$

In contrast, $A \overset{\text{a.e.}}{\cap} B$ means that the sets intersect in positive mass.

Linear Algebra

In a real vectorspace \mathbf{V} , say that

$$\dagger: \quad \sum_{j=1}^N \alpha_j \mathbf{v}_j \quad (\text{with each } \alpha_j \in \mathbb{R})$$

is a *linear combination* (*lin.comb*) of vectors (points) $\mathbf{v}_1, \dots, \mathbf{v}_N$. If, further, these scalars satisfy

$$\ddagger: \quad \alpha_1 + \alpha_2 + \dots + \alpha_N = 1,$$

then we call (\dagger) a **weighted average** of the points. Finally, if (\ddagger) and each $\alpha_j \geq 0$, then we call (\dagger) a **convex average** of the points.

Given a set $S \subset \mathbf{V}$ of points, we define three supersets

$$\text{Spn}(S) \supset \text{AffSpn}(S) \supset \text{Hull}(S).$$

The **span** is the set of all lin.combs (\dagger) , as $\{\mathbf{v}_1, \dots, \mathbf{v}_N\}$ ranges over *all finite* subsets of S . The **affine span** is the set of all (\dagger) satisfying (\ddagger) , whereas the **hull** is the smaller set of all convex averages. Thus $\text{Spn}(S)$ is the smallest *subspace* (that includes S) whereas $\text{AffSpn}(S)$ is the smallest *affine-space* and $\text{Hull}(S)$ is the smallest *convex set*.

A point $\mathbf{w} \in C$ is an “**extreme point** of a convex set C ” if: *Whenever we write $\mathbf{w} = \alpha_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2$ as a convex average (of points $\mathbf{v}_1, \mathbf{v}_2 \in C$), then necessarily $\mathbf{v}_1 = \mathbf{v}_2 = \mathbf{w}$.* A non-void set $C \subset \mathbf{V}$ is an N -dimensional **simplex** (an “ N -simplex”) if we can write it as

$$C = \text{Hull}(\mathbf{w}_1, \dots, \mathbf{w}_{N+1})$$

where no \mathbf{w}_j is in the affine-span of the others. Equivalently, C has precisely $N+1$ extreme-pts, and $\text{Dim}(C) = N$.

Polynomials

Use **poly** for “polynomial”. An integer-coefficient poly is a \mathbb{Z} -poly or an **intpoly**. With rational coeffs, it is a \mathbb{Q} -poly or **ratpoly**. An \mathbf{F} -poly has its coeffs come from a **field** \mathbf{F} . (A commutative ring is ok too).

The poly **zip** has all of its coefficients zero. Say that a poly is **5-topped** if its degree is *strictly* less than 5. Over a field \mathbf{F} , the set of (single variable) N -topped polys forms an N -dimensional *vectorspace*.

Counting

For a natnum n , use “ $n!$ ” to mean “ n **factorial**”; the product of all positive-integers less-equal n . So $3! = 3 \cdot 2 \cdot 1 = 6$ and $5! = 120$. Also $0! = 1$ and $1! = 1$.

The **binomial coefficient** $\binom{7}{3}$, read “7 choose 3”, means *the number of ways of choosing 3 objects from 7 distinguishable objects*. If we think of putting these objects in our left pocket, and putting the remaining

4 things in our right pocket, then we write the coefficient as $\binom{7}{3,4}$. [Read as “7 choose 3-comma-4.”] Note that $\binom{7}{0} = \binom{7}{0,7} = 1$. Also note this identity:

$$[x + y]^N = \sum_{j+k=N} \binom{N}{j,k} \cdot x^j y^k,$$

where (j, k) ranges over all *ordered* pairs of natural numbers with sum N .

In general, for natnums $N = K_1 + \dots + K_L$, the **multinomial coefficient** $\binom{N}{K_1, K_2, \dots, K_L}$ means the number of ways of partitioning N different things, by putting K_1 of them in pocket 1 and K_2 of them in pocket 2, and so on. Easily

$$\binom{N}{K_1, K_2, \dots, K_L} = \frac{N!}{K_1! \cdot K_2! \cdot \dots \cdot K_L!}.$$

And $[x_1 + \dots + x_L]^N$ indeed equals the sum of

$$\binom{N}{K_1, \dots, K_L} \cdot x_1^{K_1} \cdot x_2^{K_2} \cdot \dots \cdot x_L^{K_L},$$

taken over all natnum-tuples $\vec{K} = (K_1, \dots, K_L)$ that sum to N .

Let “50 **placed into 3 types**” (or just “50 into 3 types”) denote the number of ways of choosing 50 individual candies from 3 distinct types of candy. Write it $\left[\begin{smallmatrix} 50 \\ 3 \end{smallmatrix} \right]$. For $N, T \in \mathbb{N}$, use $\left[\begin{smallmatrix} N \\ T \end{smallmatrix} \right]$ for “ N (objects) into T -types”, the number of ways of filling T distinct types with N objects total. So $\left[\begin{smallmatrix} 0 \\ T \end{smallmatrix} \right] = 1$ for each natnum T . And $\left[\begin{smallmatrix} N \\ 0 \end{smallmatrix} \right] = 0$ for each *posint* N . Furthermore

$$\forall T \in \mathbb{Z}_+ \quad \cdot \quad \left[\begin{smallmatrix} N \\ T \end{smallmatrix} \right] = \binom{N + T - 1}{N, T - 1} = \left[\begin{smallmatrix} T - 1 \\ N + 1 \end{smallmatrix} \right].$$

This, since $\binom{N+T-1}{N, T-1} = \binom{T-1 + [N+1]-1}{T-1, [N+1]-1}$.

Number Theory

Use \equiv_N to mean “congruent mod N ”. Let $n \perp k$ mean that n and k are co-prime. Use $k \spadesuit n$ for “ k divides n ”. Its negation $k \nspadesuit n$ means “ k does not divide n .” Use $n \spadesuit k$ and $n \nspadesuit k$ for “ n is/is-not a multiple of k .” Finally, for p a prime and E a natnum: Use double-verticals, $p^E \spadesuit n$, to mean that E is the **highest** power of p which divides n . Or write $n \spadesuit p^E$ to emphasize that this is an assertion about n . Use **PoT** for Power of Two and **PoP** for Power of (a) Prime.

A natnum N is a **SOTS**, Sum Of Two Squares, if there are integers for which $\ell^2 + k^2 = N$. If there exists such a pair with $\ell \perp k$, then N is **coprime-SOTS**. (E.g, 25 has a non-coprime rep as $5^2 + 0^2$; nonetheless, 25 *is* coprime-SOTS, since $25 = 3^2 + 4^2$. OTOHand, both $4 = 0^2 + 2^2$ and $40 = 2^2 + 6^2$ have these unique SOTS reps, so neither is coprime-SOTS.) An odd integer L is **4Neg** if $L \equiv_4 -1$ and is **4Pos** if $L \equiv_4 +1$. Fermat's Prime-SOTS Thm says: *Oddprime p is SOTS iff p is 4Pos.*

Mod N , a **rono** is a (square-)Root Of Negative One; an integer I such that $I^2 \equiv_N -1$.

Use **CRT** for the Chinese Remainder Thm.

For N a posint, use $\Phi(N)$ or Φ_N for the set $\{r \in [1..N] \mid r \perp N\}$. The cardinality $\varphi(N) := |\Phi_N|$ is the **Euler phi function**. (So $\varphi(N)$ is the cardinality of the multiplicative group, Φ_N , in the \mathbb{Z}_N ring.) Use **EFT** for the Euler-Fermat Thm, which says: *Suppose that integers $b \perp N$, with N positive. Then $b^{\varphi(N)} \equiv_N 1$.*

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