Reading

Have a sheet of paper called something like Questions/Ideas. When reading the text, when doing homework, have this sheet convenient and write down things which puzzle you, ideas you have (“Will this shortcut work?”, “Is this step valid?”)

When reading an example: Write the given problem down, close the text, and work hard on the problem for twenty-or-so minutes. Compare your approach with that of the text. If you find an error in the text, write it down on Questions/Ideas.

Writing

Write in complete sentences. Each sentence should start with a capital letter and end with a period. A good way to get the hang of this is to read sentences from textbooks out loud. This way you force yourself to pronounce math symbols and give you the facility to put math into sentence form.

If you introduce a new letter, write down a phrase saying what the letter means. For example “Let $d$ denote the distance from the centroid to the line.” Be specific! Here is a better version of the preceding sentence:

“Let $d$ denote the perpendicular distance from the centroid of the region $R$ to the line $L$.”

Sometimes, a carefully drawn and labeled picture can help define a quantity.

Be specific

In an integral, always put in the integration variable: Not $\int y^x$, but rather $\int y^x \, dx$ or $\int y^x \, dy$, as the case may be. For summations, write in the summation index: Write $\sum_{n=3}^{\infty} a_n$ and not just $\sum a_n$.

If you are computing a definite integral and write $\int_3^7 h(x) \, dx$, don’t forget to write the bounds of integration as you continue your computation. If you prefer to compute an antiderivative and then plug in bounds later, tell me so: “Let’s compute the indefinite integral $\int h(x) \, dx$ as follows.”

Do not overuse vague words like “it”. Poor: “Plug it into that.” Better: “Plug the number into the function.” Better still: “Plug $x = 2$ into $h'(x)$” or “Plug 2 into the derivative of $h$.”

Announce the methods you use: “Using integration by parts, I compute…” “I now check that l’Hôpital’s Rule may be applied…”

Announce your result: If the question asks you to compute (or define) such-and-such, your writeup should have a sentence saying explicitly “Such-and-such equals (is defined to be)…” (Don’t just say “the answer is…” because sometimes it is not clear which (part of) the question your answer is an answer to. Be specific.)

Note: You will find it easier to write in complete sentences by learning phrases such as “the second derivative of”, “the integrand”, “the bounds of integration”, “the summand”, “the index of summation”, “polynomial”, “coefficient”.

Studying

Reading alone will never give proficiency. Write up many problems and keep your solutions in a notebook. [Updated: Post your problem+soln to our class Archive.] If your solutions are well-explained, then this notebook [Archive posting] will become a
valuable study aid. I generally recommend writing up between 20 and 30 Calculus problems per chapter.

**Create your own problems.** Can you make a series problem that you can solve, but your classmate can’t? Can you make a problem that you can solve in three different ways? Invent a solid-of-revolution problem that can be solved by both the shell and disk method’s. Create a solid-of-revolution problem where the solid is rotated about an axis which is neither vertical nor horizontal. Create a series problem where the Ratio Test is inconclusive but the Limit Comparison Test is conclusive. *If you can create interesting problems in a subject, you are starting to understand the subject.*

**Talk over problems and theorems with other people.** Try to explain a theorem (*not just state it, explain it*) to someone else. This is an excellent test of your understanding. Challenge your friend to ask you a question about the theorem that you can’t answer. Show him wrong.

**Check out two or three calculus books from the library.** Look at some of the older, thin, calculus books. Can you turn to the exercises and solve some of the problems?

**Memorize theorems.** Suppose, every day for a week, upon waking you take a piece of paper and write down the following. The definition of: the inverse of a function; limit of a function, continuity, derivative; the natural logarithm and exponential functions; limit of a sequence, limit of a series; centroid of a region. As well as: $n$ factorial, “$n$ choose $k$” (for what values of $n$ and $k$ does that definition work?), the $n,k$ binomial coefficient. And you write the complete statements of: The Fundamental Theorem of Calculus, l’Hôpital’s Rule, The trapezoid and Simpson’s Rule, Taylor’s theorem, The Binomial theorem and Binomial Series theorem.

And you draw the *pictures* for: The Shell Method, The Disk Method, Theorem of Pappus. Note that the *formulas* only handle special cases (when the axis of rotation is vertical etc.). But the *pictures* always work.

After a week, you’ll have these notions at your fingertips!

**Taking exams**

Take a sheet of paper and write down the complete statements of the theorems you think you will need. This is a “crib sheet” from your own brain!

Work methodically, rather than chaotically. It is faster.