

All four HW problems were emailed out; I have typeset the 4<sup>th</sup>-problem, here, to make it easier to read. I suggest that you staple this sheet as first page of your soln to problem 4.4.

Essays should abide by the CHECKLIST on  
<http://www.math.ufl.edu/~squash/teaching.html>

It will help me if you write your ordinal, LARGE, in the upper-RH corner of the first page of your WU, and your name below or next to it. Ta.

**4.4:** We work modulo  $M := 191$ , which is prime. Its multiplicative-group,  $\Gamma$ , has  $\varphi(191)=190$  elements. This  $\Gamma$  is cyclic, and  $G := 19$  is a generator, i.e  $\text{Ord}_\Gamma(G) = 190$ .

Use BSGS (“Baby-Step Giant-Step”) to compute the unique exponent  $E$  in  $[0 .. 190)$  for which

$$19^E \equiv_M 23.$$

**a** Draw a large circle-picture and label the entries of the bottom-right patch by  $G^0 \equiv 1$ ,  $G^1 \equiv 19$ ,  $G^2 \equiv 170$ , ... up to  $G^{12} \equiv ??$ , putting in the actual values. [*Optional:* Produce a sorted version of this list, for binary searching.]

**b** Draw in the other patches; how many are there? By how much does our last patch overlap our initial patch?

**c** What is the value of the multiplier, call it  $U$ , which carries us back to the previous patch? Now use BSGS to compute the above  $E$ . Which patch was it in?

**d** Use repeated-squaring to check that your value for  $E$  is correct.