Ergodic HW

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H1: Suppose $(T : X, \mathcal{X}, \mu)$ is a bi-mpt. and A is a (measurable) set such that $T^{-1}(A) \stackrel{\text{a.e}}{=} A$. Prove that there exists a set $B \stackrel{\text{a.e}}{=} A$ so that B is pointwise invariant, i.e, $T^{-1}(B) = B$.

H2: Let $R = R_\alpha$ be an irrational rotation of the circle $X := [0, 1)$. Partition the circle into two half-open intervals, say $A := [0, \frac{1}{3})$ and $B := [\frac{1}{3}, 1)$. Given a point $z \in X$, let the ***forward name*** of z be the sequence $(s_0, s_1, s_2, \dots)$, where s_n is the symbol "A" or "B" as $R^n(z)$ is in A or B.

Use the density of forward (or reverse) orbits to prove that distinct points have distinct forward names. What does this say about predictability?