Probability II
MAP6473 4810

Homework-2.

Prof. JLF King
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Reading. Please read examples 8.1 and 8.2 on P.112 of Billingsley, as well as 8.3 and 8.4. Note that Billingsley's MChains may have *countably*-many states; hence there need not be an invariant distribution.

Notation in force. Fix a positive integer $\mathfrak{D}$. Let $\mathbb{P} = \mathbb{P}^{\mathfrak{D}-1}$ be the simplex of probability vectors $\mathbf{v} \in \mathbb{R}^{\mathfrak{D}}$. Fix a $\mathfrak{D}\times\mathfrak{D}$ ***(column)-stochastic*** matrix M; each column is a probability vector.

2.1: Let $K_0 := \mathbb{P}$. For each posint n let

$$K_n := \mathsf{M}^n(\mathbb{P}) \overset{\text{note}}{=\!=} \mathsf{M}(K_{n-1}).$$

Then $\Lambda := \bigcap_0^\infty K_n$ is compact and non-void.

Prove that $\mathsf{M}()$ maps Λ <u>into</u> itself (not necessarily properly).

Since each K_n is convex, so is Λ. Prove or disprove: Λ is a simplex.

Give an example where

$$\mathbb{P} \supsetneqq \Lambda \supsetneqq \text{FixPoint}(\mathsf{M}).$$

By FixPoint(M) I mean the set of M-invariant probability vectors.

2.2: With Λ as above: Prove or disprove: The M mapping sends Λ <u>onto</u> itself.

Optional. (no points, other than brownie points) For those who like Topology: Suppose $f{:}X\circlearrowleft$ is a continuous map on a compact metric space. Let $\Lambda := \bigcap_{n=0}^\infty f^n(X)$. Give sufficient and necessary conditions on (X, f) for f to map Λ <u>onto</u> Λ. Give examples of various possibilities.