

(Opt) Project-H

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Touch: 2Apr2017

Notes. This project is optional. It can raise or lower your estimated grade by one click.

Pamphlet *Chromatic polynomial of a graph* on our TEACHING PAGE has definitions. If possible, please use the notation from that pamphlet.

Use $u-v$ to mean that vertices u and v have an edge between them.

Due by **3PM, on Tuesday, 23Apr2013** slid completely under my office door.

OYOP: Your 3 essay(s) must be TYPESET, and Double or Triple spaced. Use the *Print/Revise* cycle to produce good, well thought out, essays. Start each essay on a *new* sheet of paper.

Do **not** restate the problem; just solve it.

H1: For natnum J and tuple $\vec{b} = (b_1, b_2, \dots, b_J)$ of posints, a graph is **\vec{b} -nice** if its chromatic polynomial is

$$*: f_{\vec{b}}(x) := x \cdot [x - 1]^{b_1} \cdot [x - 2]^{b_2} \cdots [x - J]^{b_J}.$$

Let \mathcal{W} be the set of nice graphs.

a Give an algorithm which, given a \vec{b} , explicitly constructs a \vec{b} -nice graph $S_{\vec{b}}$.

The number of edges in $S_{\vec{b}}$ is $\sum_{n=1}^J \dots$.

b When $J := |\vec{b}| = 1$, prove that each \vec{b} -nice graph, H , is a tree.

c Prove or CEX: “Collection \mathcal{W} is sealed under (basic) full-product.”

d Prove or CEX: “Each nice graph with $J \geq 2$, is chromatically unique.”

e Can you characterize the nice graphs?

H2: Prove there is no graph S whose chromatic polynomial is

$$h(x) := x \cdot [x^5 - 5x^4 + 12x^3 - 10x^2 + 3x - 1],$$

using results from our *Chromatic polynomial* pamphlet, and from Bona’s text.

H3: *Invent* an interesting non-trivial problem involving either chromatic polynomial/number, or matchings, or planar graphs/Euler characteristic. Solve the problem. Use pictures where appropriate. [Hint: Do not use a solved problem from Bona’s text.]

End of (Opt) Project-H

H1: _____ 135pts

H2: _____ 55pts

H3: _____ 65pts

Total: _____ 255pts

Please PRINT your name and ordinal. Ta:

Ord: _____

HONOR CODE: “I have neither requested nor received help on this exam other than from my professor.”

Signature: _____