Individual Project
MAA4103

ACES-H

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31Aug2015

Please TYPE triple–spaced, grammatical, essays solving the problems. Your individual-project is due by **3:30PM, Friday, 24Apr2009**, slid *completely under* my office door, LIT402.

Essays violate the CHECKLIST at *Grade Peril*...

H1: Let $\mathbb{G}$ be the set of fncs $h:[0,\infty)\to\mathbb{R}$ s.t for each $z>0$, the restriction $h|_{[0,z]}$ is $\mathbb{RI}$; such an h is ***locally integrable***. Let J_z denote the $[0,z]$ interval.

α Suppose a locally-integrable f has $\lim_{x\to\infty} f(x) = 5$. For each $z>0$, let

$$\mathbf{A}_z = \mathbf{A}_z(f) := \frac{1}{z}\int_0^z f(t)\,\mathrm{d}t\,,$$

the ***average value*** of f on J_z.

Prove that $L := \lim_{z\to\infty} \mathbf{A}_z$ exists. Prove that $L = 5$.

[*Hint:* Nobody said that f was continuous.] Use pictures to illustrate your argument.

β Produce, with proof, a e.P.L (extended piecewise linear) fnc $g:[0,\infty)\to\mathbb{R}$ (continuous, hence automatically locally-integrable) such that $[\lim_{z\to\infty} \mathbf{A}_z(g)] = 5$, **yet**

$$[\limsup_{x\to\infty} g(x)] = +\infty \quad\text{and}\quad [\liminf_{x\to\infty} g(x)] = 2\,.$$

Draw a labeled, captioned graph of your g. Use pictures to illustrate your proof that your g satisfies the above three conditions.

H2: Letting $J := [0,1]$, we study fncs $J\to\mathbb{R}$. A ***step function*** is a finite linear-combination of indicator-fncs of intervals.

i For a step fnc $h := \sum_{k=1}^{K} \alpha_k \mathbf{1}_{I_k}$ [where $\alpha_k \in \mathbb{R}$ and each I_k is an interval] and an $\varepsilon>0$, construct a piecewise-linear (P.L fncs are automatically cts) fnc $\varphi() \leqslant h()$ such that $\int_J h \leqslant \varepsilon + \int_J \varphi$. [*Sugg:* As a LEMMA, prove the K=1 case. Now use the lemma to prove the general THM, arguing *carefully*. For both, draw pictures to illustrate the ideas.]

ii Prove: [Use pics to illustrate your rigorous argument.]

1: Continuous-are-nearby thm. *Fix $f \in \mathbb{RI}(J\to\mathbb{R})$. Given $\varepsilon>0$, there exists a continuous function $\theta()$, with $\theta \leqslant f$, such that $\int_J f \leqslant \varepsilon + \int_J \theta$.* ◊

iii Use pictures to show how *your construction* works when $f := -\mathcal{R}_{\mathbb{D}}$, the negative of the Ruler-fnc of the dyadic rationals.

H3: Show no work. Fill-in every *blank* on **this** sheet; handwritten is fine.

a For each $K \in \mathbb{R}_+$:

$$\lim_{n\to\infty} \frac{1^K + 2^K + 3^K + \cdots + n^K}{n^{K+1}} = \underline{\qquad\qquad}.$$

[*Hint:* Interpret each "limitand" as a Riemann-sum.] Also,

$$\lim_{n\to\infty} \left[\tfrac{1}{2n+1} + \tfrac{1}{2n+2} + \tfrac{1}{2n+3} + \cdots + \tfrac{1}{5n}\right] = \underline{\qquad\qquad}.$$

b The closest point-pair, P on parabola $y = x^2 + 27$ and Q on the line through the origin with slope 3, is

P=(____ , ____), Q=(____ , ____).

End of ACES-H

H1:	___ ___	65pts
H2:	___ ___	85pts
H3:	___ ___	45pts
Poorly stapled, or missing ordinal :	___	−5pts
Missing name, or honor sigs :	___	−5pts
Poorly proofread:	___ ___ ___	−One Million pts
Total:	___ ___ ___	195pts

Print name .... Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....