Staple!

Ord: ___ ___

Combinatorics MAD4204 4563 — **Project-G** — Prof. JLF King, Thursday, 19Apr2018

Due **2PM, Thursday, 26Apr2018**, *slid completely under my office door*, LIT 402. *This sheet is* "Page $1/N$"; you've labeled the rest as "$2/N$", ..., "N/N".

And: For a vertex w of a graph, use $\mathrm{Nbr}(w)$ for the neighborhood of w.

You may use without proof: *If* $0 \leqslant b < c \leqslant \frac{N}{2}$, *then* $\binom{N}{b} < \binom{N}{c}$. *Also* $\binom{N}{N-k} = \binom{N}{k}$.

G1: *Show no work.*

The vertex set of H_N is $\mathbb{V} := [1..3N]$. For $\mathbf{u} \in \mathbb{V}$, *when possible*: $\mathbf{u}$—$[\mathbf{u}+3]$. *If* $\mathbf{u} \equiv_3 1$, *then* $\mathbf{u}$—$[\mathbf{u}+1]$ *and* $\mathbf{u}$—$[\mathbf{u}+2]$. *If* $\mathbf{u} \equiv_3 2$, *then* $\mathbf{u}$—$[\mathbf{u}+1]$. Then

$\mathcal{P}_{H_N}(x) =$ ....

In grammatical English sentences, TYPE *each essay on every* 2^nd^ *line* (usually), *so that I can write between the lines.* (Do **not** restate the Q.) *Start each essay on a new page.*

G2: A graph's automorphism group is ***transitive*** if for all vertices u,w, *there exists* automorphism α with $\alpha(u) = w$.

Exhibit a (finite, simple) connected vertex-regular graph, G, whose $\mathrm{Aut}(G)$ is *not* transitive, with proof.

Construct such a G which is *planar*.

Produce such a 3-regular ***rigid*** [only the *Id* aut] graph.

[Warning! A "face" is **not** a property of a graph; it is a property of a graph-*embedding*.]

G3: Fix posints $\mathbf{K},\mathbf{N}$ with $\mathbf{K} < \frac{\mathbf{N}}{2}$, and $\Omega := [1..\mathbf{N}]$, the ***token set***. Let $\mathcal{P}_m$ be the collection of m-token subsets of Ω. Define a bipartite graph $R = R_{\mathbf{K},\mathbf{N}}$ on sets $\mathcal{B} := \mathcal{P}_{\mathbf{K}}$ and $\mathcal{G} := \mathcal{P}_{\mathbf{K}+1}$, with a boy-girl edge b—g IFF $\mathsf{b} \subset \mathsf{g}$.

i. Prove that $\mathcal{B}$ satisfies Hall's condition. Thus, $\exists$ a ***good*** injection $\varphi: \mathcal{B} \hookrightarrow \mathcal{G}$ with b—$\varphi(\mathsf{b})$, for every $\mathsf{b} \in \mathcal{B}$.

Call such an injection, a **"$(\mathbf{K},\mathbf{N})$-*good-map*"**.

ii. Exhibit a specific $(2,5)$-good-map.

iii. Construct a specific $(\mathbf{K}, 1+2\mathbf{K})$-good-map.

iv. Construct a specific $(\mathbf{K},\mathbf{N})$-good-map. Bóna **26** *P.293*.

G4: G is a [finite, connected] planar multigraph *embedding*, where each face is a 2-gon, 3-gon or 4-gon. Let p_2, p_3, p_4, be the numbers of such faces. Just as in an octahedron, suppose each vertex has degree=4, and that $p_2 + p_3 = 8$.

Prove that $p_2 = 0$. Bóna **22** *P.316*.

Exhibit/describe such an embedding with $\geqslant 99$ vertices.

G5: From chapter 9, 11, 12 or 13 of Bóna's text, pick one interesting supplementary problem and solve it nicely.

End of Project-G

G1:	___ ___	20pts
G2:	___ ___	35pts
G3:	___ ___	85pts
G4:	___ ___	35pts
G5:	___ ___	35pts
Total:	___ ___ ___	210pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
Name/Signature/Ord

Ord: ....