ACES MAA4103 | **Prac-G** | Prof. JLF King, Monday, 06Apr2009

Hello. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

G1: Show no work.

z) In $\mathbb{R}$: $[1+\boldsymbol{i}]^{2009} = [\ldots] + \boldsymbol{i}\cdot[\ldots]$.

[*Hint: Multiplying complexes multiplies their moduli, and adds their angles. You may use* sin *and* cos *if you wish.*]

z2) Sqroot of $\boldsymbol{i}-1$ in the upper-halfplane is $r\cdot\text{cis}(\theta)$, where $r=$ ______ $\in\mathbb{R}_+$ and $\theta=$ ______ $\in[0,\frac{\pi}{2}]$.

[*Hint:* Recall, for $\theta\in\mathbb{R}$, that $\text{cis}(\theta)$ abbreviates the sum $\cos(\theta)+\boldsymbol{i}\cdot\sin(\theta)$.]

a) In $\mathbb{R}$, the set, $\mathbb{I}$, of irrational numbers is a $\mathcal{G}_\delta$-set. We know this because

we can write $\mathbb{I} =$ ______.

b) Poly $\beta(x) := x^{19} + x^{96}$ has 9^{th} derivative,

$\beta^{(9)}(x)=$ ______. (Coeffs ITOf prods and quotients of factorials.)

Our integral-formula of the 9^{th} Remainder-term, centered at 5, evaluated at 4, is

$$\mathbf{R}^{\beta}_{9,5}(4) = \int_{\ldots}^{\ldots} \ldots \cdot dt.$$

c) Let $h := [y \mapsto \cos(2y)]$. Then the 5-topped poly $\mathbf{T}^h_{5,\pi}(x)=$ ______.

[*Hint:* The center of expansion is π, not zero.]

d) P.L fncs $f_n \xrightarrow[n\to\infty]{\text{ptwise}} \mathbf{0}$ have $[\int_0^5 f_n] = n^2$. The cutpoint and height tuples of f_n are

$\vec{\mathbf{p}}_n := (0,$ ______ $,$ ______ $,5)$

and $\vec{\mathbf{h}}_n := (0,$ ______ $,$ ______ $,0)$.

And $\|f_n\|_{\sup} =$ ______.

e) Map $g\colon[0,1]\to\mathbb{R}$ is *not* RI, yet $|g|$ *is* RI, where $g :=$ ______.

f) Writing poly $p(x) := 6 + 23x^2 - 7x^3 + 4x^4$ as $\sum_{k=0}^{4} C_k\cdot[x+2]^k$, coeff C_3 is in: Circle *one* interval

$(-\infty,-70)$, $[-70,-15)$, $[-15,-8)$, $[-8,-1)$, $[-1,8)$, $[8,15)$, $[15,30)$, $[30,75)$, $[75,94)$, $[94,+\infty)$.

g) Let $\varphi(x,y) := x^3y^5 + 2^x$. Then the Hessian matrix of φ is $H(x,y)=$ ______ (More room than given here.)

h1) On the circle $x^2+y^2 = 1^2$, the max-point of $\Gamma(x,y) := \sin(xy)$ is (______ , ______).

h2) On the ellipse $[\frac{x}{3}]^2 + [\frac{y}{4}]^2 = 1^2$, the max-point of $\Gamma(x,y) := x - 2y$ is (______ , ______).

i) An explicit bijection $F\colon\mathbb{N}\hookrightarrow\mathbb{Z}$ is this:

When n is *even*, then $F(n) :=$ ______.

When n is *odd*, then $F(n) :=$ ______.

j) Let $J := [0,1]$. A map $h\colon J\to\mathbb{R}$ is ***Lipschitz cts*** IFF

______

______

An example of a *continuous* but **not** Lipschitz $f\colon J\to\mathbb{R}$, which is *differentiable* on $J^\circ = (0,1)$ is

$f(x) :=$ ______.

Essay question: Carefully write a triple–spaced essay solving the problem.

G2: Let $J := [0,1]$ and $K := [3,5]$. Suppose $g\colon K\to\mathbb{R}$ is Lipschitz cts, with Lipschitz-constant 7. Suppose $f \in \mathbb{RI}(J\to K)$. Let $h := g\circ f$. Prove that h is integrable. [*Hint:* Start with "PROOF: Fix $\varepsilon > 0$." Perhaps define some other quantities. Now prove, given an arbitrary ptn P, that $\text{Osc}^h(\mathsf{P}) \leqslant \varepsilon$.]

G3: Set $J := [4,7]$. *Recall:* Thm A. *Fnc* $f\colon J\to\mathbb{R}$ is RI IFF $\forall\varepsilon$, $\exists$ a partition P such that $\text{Osc}^f(\mathsf{P}) \leqslant \varepsilon$.

Use Thm A to prove: Thm B. *Suppose* $f\colon J\to\mathbb{R}$ *is strictly-increasing. Then* f *is* RI.

G4: Let $J := [0,\frac{1}{5}]$. Give a specific example of a *bounded* fnc $\varphi\colon J\to\mathbb{R}$ which is *not* Riemann integrable. Give a number $L :=$ ______ >0 st. for each ptn P of J: $\text{Osc}^\varphi(\mathsf{P}) \geqslant L$. Give a *formal proof* of this assertion. [*Hint:* Do not restate the problem. Firstly, tell me what your L is, and why. Now, for each closed subinterval $B\subset J$, *prove* an appropriate lower bound for $\text{Var}^\varphi(B)$. Now...]

G5: Carefully state the version of FTC from our NOTES. [*Hint:* Can you prove it?]

G6: Prove: *Let* $J := [0,1]$. *Suppose* $f,g \in \mathrm{RI}(J \to \mathbb{R})$. *Suppose* $|f| \leqslant 3$ *and* $|g| \leqslant 4$. *Then* $f \cdot g \in \mathrm{RI}$. [*Hint:* You'll want to explicitly use the bounds 3 and 4.]

G7: Let $J := [0,1]$. Suppose $f\colon J \to \mathbb{R}$ is RI. Prove that $|f|$ is RI. Prove that $\left|\int_J f\right| \leqslant \int_J |f|$.

End of Prac-G