Staple!

Ord: ___ ___

Combinatorics
MAD4204 4563

Class-G

Prof. JLF King
Wedn, 11Apr2018

End of Class-G

G1:	___ ___	85pts
G2:	___ ___	65pts
Total:	___ ___ ___	150pts

Please PRINT your name and ordinal. Ta:

Ord: ....

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....

Welcome. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

G1: Short answer. Show no work.

a) Prof. K will be teaching the undergrad NUMBER THEORY course this Summer-B, 2018. Also, in Spring 2019 he'll run the NUMBER THY AND MATHEMATICAL CRYPTOGRAPHY course, [**MAT4930**, section# TBD]. Circle:

Verily **Factual** 1 **Not-fake-news**

b) Having 10 triangular faces, G is an embedding of an N-vertex graph on a sphere, where $N=$ ....

c) The girls' prefs are:

$$G1\begin{bmatrix}B1\\B2\\B3\\B4\end{bmatrix},\quad G2\begin{bmatrix}B2\\B3\\B4\\B1\end{bmatrix},\quad G3\begin{bmatrix}B1\\B2\\B3\\B4\end{bmatrix},\quad G4\begin{bmatrix}B3\\B4\\B1\\B2\end{bmatrix}.$$

The boys' prefs are:

$$B1\begin{bmatrix}G4\\G2\\G3\\G1\end{bmatrix},\quad B2\begin{bmatrix}G3\\G4\\G2\\G1\end{bmatrix},\quad B3\begin{bmatrix}G2\\G3\\G4\\G1\end{bmatrix},\quad B4\begin{bmatrix}G4\\G2\\G3\\G1\end{bmatrix}.$$

A particular stable-matching is

$B1\leftrightarrow$ ...... , $B2\leftrightarrow$ ...... , $B3\leftrightarrow$ ...... , $B4\leftrightarrow$ ....

d) There are .... digraphs on vertex-set $[1..N]$.

The number of ***tournaments*** (*complete* diagraphs) on $[1..N]$ is ....

e) Below, N is the # of vertices. *For each* $N \geqslant 5$:

$\exists$ *N-graph with exactly N spanning-trees.*	T	F
$\exists$ *N-graph with exactly $N-1$ spanning-trees.*	T	F
$\exists$ *N-graph with exactly $N-2$ spanning-trees.*	T	F

OYOP: *In grammatical English* ***sentences****, write your essay on every* ***third*** *line* (usually)*, so that I can easily write between the lines.*

G2: H is a bipartite graph on Boy/Girl sets $\mathcal{B}\cap\mathcal{G}=\varnothing$, with $|\mathcal{B}|\leqslant|\mathcal{G}|<\infty$, and each boy knows at least one girl. Moreover, for all vertices $y\in\mathcal{B}$ and $x\in\mathcal{G}$, ***if*** y — x (they are connected by an edge), ***then*** $\text{Deg}(y)\geqslant\text{Deg}(x)$.

Prove $\exists$ a perfect matching for the boys. I.e, prove $\exists$ an <u>injection</u> $f:\mathcal{B}\hookrightarrow\mathcal{G}$ such that $[\forall b\in\mathcal{B}: b$ — $f(b)]$.