Finishing the semester in Style

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Please type up (preferred) or write up (acceptable) your solutions to the following problems, each solution on a separate sheet (or sheets) of paper. Use complete English sentences which are grammatical and correctly spelled. Then staple all your sheets together with *this sheet* as the top sheet. Diagrams may be helpful. You may use any books, calculators or computers, that you wish. The only human you may consult is me.

Problems stated with a page number come from our textbook: **Applied Combinatorics**, by Fred Roberts.

I am as much interested in your *explanation* as in your answer. Cite theorems you use by text and page number (or by theorem name).

You can make my grading easier by judiciously using the symbols ":=" (is defined to be) and "=" (equals). The expression "$w :=$ (something)" means that *you* are giving a meaning to the new symbol "w". Your writeup is due by 6PM on *Monday, May 1, 1995.* (Please slide your writeup under my office door, 402 Little Hall.)

β1: Let f_n denote the number of *legal* strings which consist of n left-parentheses and of n right-parentheses in some order. Let p_n denote the number of *primitive* legal expressions, as defined in class.

For $n \geqslant 1$, write the expression f_n as $A - B$, where A and B are each binomial coefficients. Here, A will count the total number of expressions (of n pairs of parentheses in some order), whether legal or not, and B will count the number of "bad" expressions –those which are not legal. Count B by the "mirror principle", as shown in class.

Can you generalize this result?

β2: On P.243, do #11. What does C_0 equal? Get a recurrence for $\{C_n\}_{n=0}^{\infty}$ (Pictures will likely be useful here.)

Here is one possible way to a recurrence; there are others: Given $2[n+1]$ points on the circle, fix one of them and call her Kim. Kim can be connected by a chord to some of the other points, splitting the remaining points into two sets; call one Cherelle and the other Frank. Consider all ways of connecting the Cherelle-points with non-overlapping chords. Consider all ways of connecting the Frank-points with non-overlapping chords. This should give you a recurrence relation. For what values of n does this recurrence hold? (Hint, hint)

Let G be the ordinary generating function

$$G(x) := \sum_{n=0}^{\infty} C_n x^n$$

Use the above recurrence to get an equation that G satisfies. Show that this is the *same* functional equation satisfied by the o.g.f of the Catalan numbers. What is a "closed formula" for $G(x)$? What is C_5?

β3: On P.244, do #18 (a)–(d). [Note that the answer section gives an answer, but no solution, to (c).] Pictures may be useful.

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