

F1: Short answer. Show no work.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\}$ $\neq 0$.

a A real number which is *not* transcendental is:

arthritis astringent irrational isometric acerbic chiral
pedestrian polyhedral asthmatic algebraic tardy

b Each three sets Ω, B, C engender a natural bijection, $\Theta: \Omega^{B \times C} \leftrightarrow [\Omega^B]^C$, defined, for each $f \in \Omega^{B \times C}$, by

$$\Theta(f) := [c \mapsto [\dots \dots \dots]].$$

Its inverse-map $\Upsilon: [\Omega^B]^C \leftrightarrow \Omega^{B \times C}$ has, for $g \in [\Omega^B]^C$,

$$\Upsilon(g) := [(b, c) \mapsto [\dots \dots \dots]].$$

c Graph G has vertex-set $\mathbb{V} := \{4, 5, 6, 7, 8, 9\}$. Two vertices $\mathbf{u}, \mathbf{v} \in \mathbb{V}$ are connected by an edge IFF difference $|\mathbf{u} - \mathbf{v}|$ is prime. [Neither 0 nor 1 is prime.]

This G is Eulerian T F

d Define $G: [1..12] \rightarrow \mathbb{N}$ where $G(n)$ is the number of letters in the n^{th} Gregorian month. So $G(2) = 8$, since the 2nd month is "February". The only fixed-point of G is \dots .

The set of posints k with $G^{\circ k}(12) = G^{\circ k}(7)$ is \dots .

[Symbol $G^{\circ k}$ is the *composition- k^{th} -power* of G . So $G^{\circ 3}(n)$ means $G(G(G(n)))$].

[January, February, March, April, May, June, July, August, September, October, November, December]

e Between sets $\mathbf{X} := \mathbb{Z}_+$ and $\mathbf{\Omega} := \mathbb{N}$, consider injections $g: \mathbf{X} \hookrightarrow \mathbf{\Omega}$ and $h: \mathbf{\Omega} \hookrightarrow \mathbf{X}$, defined by

$$g(x) := 3x \quad \text{and} \quad h(y) := y + 5.$$

Schröder-Bernstein produces a set $B \subset h(\mathbf{\Omega}) \subset \mathbf{X}$ st., letting $F := \mathbf{X} \setminus B$, the fnc $\theta: \mathbf{X} \leftrightarrow \mathbf{\Omega}$ is a *bijection*, where

$$*: \quad \theta|_F := g|_F \quad \text{and} \quad \theta|_B := h^{-1}|_B.$$

For this (g, h) , the (F, B) pair is unique. Computing,

$$\theta(56) = \dots \quad \theta(137) = \dots \quad \theta^{-1}(603) = \dots$$

Essay. Hand-write your essay on paper, writing large and dark. [You may typeset it if you wish, but that is likely slower.] *Do Not Scrunch!* You can write on every 2nd-line to make your essay easier to read. The essay is written in complete sentences, correctly spelled and punctuated, and assembled into logical paragraphs.

When the essay is done, convert it to a pdf, then name it as follows: **F2.<your name>.pdf**

E.g, Rachel Stein will name her pdf file as

F2.Rachel-Stein.pdf

There are no spaces in the filename! The extension is "pdf". A hyphen is used to separate the given-name from the family-name. (You may write names in the order you are accustomed.) \square

F2: Prove that the map $f: \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{Z}_+$, with $f(k, n) := 2^k \cdot [1 + 2n]$, is *surjective*. Now prove f *injective*. [Start your *Surjectivity* argument with **Proof** and end with **QED**. The (separate) *Injectivity* argument starts/ends with **PROOF/QED**. State *specifically* what part of what theorem you are using, for each proof.]

Value $f^{-1}(928)$ equals \dots

F3: "I have neither requested nor received help on this exam other than from my professor."

F1: _____ 100pts

F2: _____ 55pts

F3: _____ 5pts

Total: _____ 160pts