Staple!

Combinatorics 2 MAD4204 | **Class-F** | Prof. JLF King *Touch*: 2Apr2017

F1: Show no work. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

Use OGF/EGF for Ordinary/Exponential Generating Fnc.

a) Coeff of x^{12} in $1/[1+x^4]^6$ is ________.

b) Suppose $G(x)$ is the OGF of seq. $\vec{\mathbf{b}} = (b_0, b_1, \dots)$, where b_n is the number of partitions of n whose parts are primes <9. Then $G(x) =$ ________.

c) With c_n the n^{th} Catalan number, then $c_3=$ ________.

The recurrence relation satisfied by $C(x)$, the OGF of $\vec{\mathbf{c}}$, is ________.

d) With $a_0 := 1$, and $a_{n+1} := n!$, the EGF of $\vec{\mathbf{a}}$ is ________.

[A "closed formula"; no summation sign.]

e) There are $\binom{K}{J}$ many [diagonal] lattice-paths from point $(0,2)$ to $(15,5)$, where $K=$ ________ and $J=$ ________.

Such a path is ***bad***, if it touches the x-axis. And $|\text{BAD}| = \binom{N}{L}$, where $N=$ ________ and $L=$ ________.

OYOP: *In grammatical English* ***sentences****, write your essay on every* ***third*** *line* (usually)*, so that I can easily write between the lines.*

F2: Let r_n be the number of ways of painting the elements of $[1..n]$, with evenly many of them *Amber*, oddly many *Blue*, and the rest *Cream*. [Each element has just one color. Note that zero is even.] So $r_3=$ ________.

i) Let $R(x) \overset{\text{EGF}}{\longleftrightarrow} \vec{\mathbf{r}}$. Write $R(x)$ as $A(x) \cdot B(x) \cdot C(x)$, for the three EGFs of sequences $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, $\vec{\mathbf{c}}$ that you explicitly define.

ii) Compute $A(x)$, $B(x)$ and $C(x)$. Compute $R(x)$.

iii) A "closed formula" for $r_n=$ ________.

Does your formula give the value for r_3 that you computed above? (Did you remember to fill-in the blank, there?)

End of Class-F

F1:	___ ___ ___	100pts
F2:	___ ___	95pts
Total:	___ ___ ___	195pts