Staple!

Team: ____

Moda
MAA4227 MAA5229

Home-E

Prof. JLF King
Touch: 6May2016

Carefully TYPE, *double-or-triple(Abraham!)–spaced, grammatical essays solving the three problems. This is due at* **BoC on Wednesday, 02Mar2011**.
Please follow the CHECKLIST *on our* Teaching *page.*

E1: Letting $J := [0,1]$, we study fncs $J \to \mathbb{R}$. A ***step function*** is a finite linear-combination of indicator-fncs of intervals.

___ ___ i For a step fnc $h := \sum_{k=1}^{K} \alpha_k \mathbf{1}_{I_k}$ [where $\alpha_k \in \mathbb{R}$ and each I_k is an interval] and an $\varepsilon > 0$, construct a piecewise-linear (P.L fncs are automatically cts) fnc $\varphi() \leqslant h()$ such that $\int_J h \leqslant \varepsilon + \int_J \varphi$. [*Sugg:* As a LEMMA, prove the $K{=}1$ case. Now use the lemma to prove the general THM, arguing *carefully*. For both, draw pictures to illustrate the ideas.]

___ ___ ii Prove: [Use pics to illustrate your rigorous argument.]

1: Continuous-are-nearby thm. *Fix $f \in \mathbb{R}\mathrm{I}(J \to \mathbb{R})$. Given $\varepsilon > 0$, there exists a continuous function $\theta()$, with $\theta \leqslant f$, such that $\int_J f \leqslant \varepsilon + \int_J \theta$.* ♢

___ ___ iii Use pictures to show how *your construction* works when $f := -\mathcal{R}_{\mathbb{D}}$, the negative of the Ruler-fnc of the dyadic rationals.

E2: Dis/Prove: There exists a *series* $\vec{\mathbf{p}} \subset \mathbb{Q}$ which is *absolutely*–$\mathbb{Q}$-convergent, yet $\mathbb{Q}$-divergent. (I.e, sum $\sum_{n=1}^{\infty} |p_n|$ is rational, yet seq $K \mapsto \left[\sum_{n=1}^{K} p_n\right]$ fails to $\mathbb{Q}$-converge.)

Prove an Interesting Lemma that is *more general* than what you need.

E3: Show no work.

___ ___ a Interval $J := [-3, \boldsymbol{\pi}]$ has ptn P with cutpoints $\{-3, 1, \boldsymbol{\pi}\}$. Define $\beta := \left[x \mapsto \sqrt[3]{x} \cdot \mathbf{1}_{\mathbb{Q}}(x)\right]$. Then

$\mathrm{Osc}^{\beta}(\mathrm{P}) =$ .... + ...

Equipping P with sample points $\{-2, \boldsymbol{\pi}/2\}$, now

$\mathrm{RS}^{\beta}(\mathrm{P}) =$ ....

___ ___ b Use $\mathbb{I}\mathrm{RI}$ for "Improper RI". Produce functions $\psi_n \in \mathbb{I}\mathrm{RI}(\mathbb{R} \to \mathbb{R})$ st. $\psi_n \xrightarrow[n \to \infty]{\text{unif}} \mathbf{0}$, yet $\left[\int_{\mathbb{R}} \psi_n\right] \not\to 0$, as $n \to \infty$. Indeed, $\forall n$: $\left[\int_{\mathbb{R}} \psi_n\right] = n^2$. For example, let

$\psi_n :=$ ....

End of Home-E

E1:	___ ___ ___	115pts
E2:	___ ___	75pts
E3:	___ ___	45pts
Poorly stapled, or missing names or honor sigs:	___ ___	−15pts
Not double-spaced:	___ ___	−15pts
Poorly proofread:	___ ___ ___	−One Million pts
Total:	___ ___ ___	235pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: ....

Ord: ....

Ord: ....